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Information Economics and Financial Economics

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by

Yilin Wang

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ABSTRACT OF THE DISSERTATION

Information Economics and Financial Economics

by

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Professor Pierre-Olivier Weill, Chair

My thesis consists of three chapters on information economics and financial economics.

Chapter 1:

Many decentralized over-the-counter (OTC) markets have recently become subject to new regulations requiring transparency. I build up an information model that features bilateral trade in a double auction, endogenous public signal, and inter-dealer network formation to study the effect of TRACE on the inter-dealer markets. In the trading stage, I study the private information diffusion process and endogenize the public information contained in the disseminated trading price. I show that in markets with a relatively low degree of information asymmetry, post-trade transparency makes the adverse selection more severe and reduces the surplus from asset reallocation between dealers, and thus hurts the inter-dealer network formation. Investors are more likely to be symmetrically uninformed about thinly traded bonds. The empirical results provide evidence that TRACE has a significant negative effect on the inter-dealer trading frequency for thinly traded bonds.

Chapter 2 (with Kim-Sau Chung):

During currency crises, large traders once simultaneously short the asset markets and currency market. We study the large trader's information manipulation in crises by introducing a large trader in an asset market and a currency-attack coordination game with imperfect information. The asset price realized in the asset market aggregates dispersed private information acting as a public signal in the currency attack game. We show that the incentive of the large trader to manipulate the asset price in favor of its currency attack leads to financial contagion. In equilibrium, the large trader's manipulating

the asset price to be lower and attacking the currency regime are concurrent; the large trader's manipulation in the asset market is most significant when the public signal is in the intermediate range. To draw policy implication regarding the market transparency, we show that when the asset market is transparent, a natural equilibrium refinement that incorporates forward induction reasoning would select the equilibrium where every trader behaves most aggressively in the currency-attack game and the currency regime is most fragile.

Chapter 3 (with Yinqiu Lu):

The way central banks manage their foreign reserve assets has evolved over the past decades. One major trend is managing reserves in two or more tranches – liquidity tranche and investment tranche – especially for those with adequate reserves. Incorporating reserve tranching, we have developed in this paper a central bank's reserve portfolio choice model to analyze the determinants of the currency composition of reserves. In particular, we adopt the classical mean-variance framework for the investment tranche and the asset-liability framework for the liquidity tranche. Building on these frameworks, the roles of currency compositions in imports invoicing and short-term external debt, and risk and returns of reserve currencies can be quantified by our structural model – a key contribution of our paper given the absence of structural models in the literature. Finally, we estimate the potential paths of the share of RMB in reserves under different scenarios to shed light on its status as an international currency.

The dissertation of Yilin Wang is approved.

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Pierre-Olivier Weill, Committee Chair

University of California, Los Angeles

2020

To my parents,

who gave me—among so many other things—strength.

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CHAPTER 1

Information Sharing Policy and Inter-dealer OTC Markets

1.1 Introduction

Many regulations aim at enhancing information sharing in financial markets. In 2002, FINRA (Financial Industry Regulatory Authority) began requiring the timely public dissemination of post-trade price and volume information for the U.S. corporate bond market through TRACE (FINRA's Trade Reporting and Compliance Engine). TRACE has become the template for increased transparency in other over-the-counter financial markets. FINRA expanded TRACE to several other asset classes, including Agency-Backed Securities, since 2010, and Asset-Backed Securities, since 2011. Title VII of Dodd-Frank Wall Street Reform has required that swaps (including CDS, interest rate swaps, collateralized debt obligations, and other derivatives) adopt TRACE-like post-trade transparency since 2011. European MiFID II/R regulations mimic TRACE for European corporate bonds and were implemented in 2018.

[ACP13] also examine the effect of TRACE on thinly traded, high-yield bonds. They find after transparency, while trading costs declined significantly for the entire bond market, there was a significant decline in the number of trades for thinly traded bonds. Many of the securities markets that are newly subject to transparency are thinly traded. Their empirical results support the view that not every segment of a security market should be subject to the same degree of mandated transparency.

This suggests that theoretical models are needed to study the non-uniform effects of transparency policy on the different segments of OTC markets. In this paper, I build up a model that features bilateral trade in a double auction between dealers, endogenous public signal, and inter-dealer network formation. My model shows that in markets with a relatively low degree of information asymmetry, post-trade transparency makes the adverse selection more severe and reduces the surplus from asset reallocation between dealers. In those markets, TRACE hurts the network formation and lowers inter-dealer trading frequency.

In the trading stage, I study a line trading network of risk-neutral dealers. For the sake of tractability, I assume the first dealer in the line network has private information about the asset payoff. The dealers have their own private values that are their private information. The trade sequentially takes place between the first dealer and the second dealer, then between the second dealer and the third dealer, etc. Each dealer chooses its demand functions for the transactions, to maximize its expected profits, given its private value and its information about the asset payoff, and they take their price impacts into consideration. I characterize the linear Bayesian Nash equilibrium and show that in equilibrium, the information asymmetry between two consecutive dealers before their trade decides how aggressively they trade on their private values, and thus decides the private information diffusion process and the surplus of asset reallocation between them.

To study the effect of TRACE on the inter-dealer markets, I introduce the public signal to the model. For simplicity, I consider the release of just one public signal. I study two different information structures of the public signal. I first show the effect of transparency on dealers' trading surplus when the public signal is exogenous. Then I show the effect of transparency on dealers' trading surplus after endogenizing the public signal as the disseminated price of TRACE. Contrast to traditional wisdom, my model shows that the disseminated trading price as the public signal is not desirable for some markets. The key difference between the endogenous and the exogenous public signal is that the endogenous public signal is both about asset payoff and about previous dealers' private values. The information about private values can allow the more informed dealer to filter the signal more effectively.

In the network formation stage, I study a network formation model to endogenize the inter-dealer network structure. The network formation stage is before the trading stage; thus, dealers trading surplus in the trading stage affects their network formation decision. In the network formation model, the dealer with private information about the asset payoff bargains with the second dealer over how to split the link formation cost;

the second dealer who forms the link bargains with the third dealer over how to split the link formation cost, etc. In equilibrium, the surplus of each formed link would spread over all the dealers before this link and affect their network formation decision. Thus the effect of TRACE on downstream dealers' trading profit would affect the upstream dealers' link formation decision. The lower trading profits of downstream dealers could hurt the the formation of the whole network. This model implies that in markets with low information asymmetry, TRACE hurts the network formation and thus lowers inter-dealer trading frequency. The network formation game also implies that TRACE could reduce the trading frequency of the upstream dealers as well, because they benefit less from the downstream dealers' trading profits if they form the link to initiate the trade.

On the empirical side, I study the effect of TRACE on corporate bond inter-dealer markets, by utilizing the Academic Corporate Bond TRACE data and the Mergent FISD database. The argument is that if a bond is very actively traded, then although investors are more likely to know more about them, they are also more likely to be asymmetrically informed; vice versa, if a bond is thinly traded, then investors are more likely to be symmetrically uninformed. So the model predicts that TRACE would have negative effects on inter-dealer trading activity for thinly traded bonds. The empirical findings in this paper show that TRACE has significant negative effect on the inter-dealer trading frequency only for thinly traded bonds. To be more specific, DID estimates show that TRACE has a non-negative or positive effect on the inter-dealer trading frequency of actively traded bonds, Phase 2 bonds, and Phase 3B bonds. In contrast, TRACE significantly lowers the inter-dealer trading frequency of Phase 3B bonds, which are the most thinly traded corporate bonds. It is a warning for the widespread implementation of TRACE, as many of the assets that are newly subject to transparency are similar to Phase 3B bonds as they are also thinly traded.

Related Literature

This paper follows the double auction literature when modeling the trading game, e.g. [Kyl89] and [MR17]. My work adds to the growing literature on network studies in financial markets. The application of network theory to financial markets has only just begun. [BEK09] and [GK07] study how a network intermediates trades in a decentralized market.

[Gof11] assesses the efficiency of resource allocation through the trading network in an OTC market. [MR17] develop a general framework for studying dealers' strategic interactions in decentralized markets. Many past studies also focus on information acquisition from a network and its impact on financial markets. [HY13] extend the rational expectation equilibrium model to study the information network in a financial market. [BK18] and [BKW19] study information transmission through inter-dealer networks in the OTC markets by extending the model in [Viv11] to games in networks. In addition to using network models to study OTC markets, others apply network models to the interbank market to analyze contagion risk in the banking system, e.g. [Bab16] and [EGJ14].

Most models of OTC markets are based on search and bargaining. [DGP05] and [DGP07] study how search and bargaining determine prices in the OTC markets. [AEW15] study how market entry costs help determine the structure of OTC trading, and thereby prices charged in OTC trading.

There is a branch of theoretical work on the impact of transparency on trading behavior in financial markets. [PR96] argue that well-informed dealers may be able to extract rents from less well-informed customers in an opaque market. [BO99] show that transparency can reduce market-makers incentives to supply liquidity, if market makers have more difficulty unwinding inventory following large traders. [NNV99] show that transparency can improve dealers' ability to share risks, which decreases their inventory costs and therefore customers' costs of trading. [Mad95] demonstrates hat dealers may prefer not to disclose trades because they benefit from the reduction in information. Some of this work highlights the downside of more transparency, but none of it studies the impact of transparency on network formation in the OTC market. My paper fills this gap in the literature.

The effect of TRACE on the US corporate bond market has been studied in some empirical work. [BMV06], focusing on Phase 1 only, which covered investment grade and large issue bonds, document a reduction in trade execution costs, estimated using a structural model. [EHP07] and [GHS07a], both using investment grade bonds in Phase 2 TRACE data, report no effect on the trading activity and a decline in transaction costs. [ACP13] find that even though trading costs decrease significantly across all types of bonds, transparency effects are not uniform across different segments of the bond market; after transparency, there is a significant decline in the number of trades for Phase 3B bonds, which are far more likely to be lower rated high yield bonds, while transparency has a limited impact on the trading activity of investment grade bonds and the most frequently-traded bonds. [BM08] survey dealers and report that bond dealers almost universally perceive that trading became more difficult after TRACE.

There is a set of studies on municipal bonds. On January 31, 2005, the Municipal Securities Rulemaking Board (MSRB) started requiring that information about trades in municipal bonds be reported within 15 minutes, similar to TRACE. Prior to that dissemination, [GHS07b] find significant price dispersion in new issues of municipal bonds, which they attribute to the decentralized and opaque market design. [Sch12] compares price dispersion at the offering date for municipal bonds before and after this change and finds that it falls sharply. [BLS17] show that the MSRB transparency rule reduced trading volume in uninsured bonds, but not in insured bonds.

Some papers study another kind of information sharing, that is, dealers' sharing of clients' bid information. [DFK19] document that equity dealers share clients' bid information with other clients. [BLV16] use a quantitative model to study the effect of dealers' information sharing of clients' order order flow information in a centralized market.

Some empirical papers study the effect of public information on information asymmetry. [KV94] and [KV97] suggest that public information releases may actually increase information asymmetry if market participants differ in their ability to interpret the news. Findings in [Gre04] indicate that the release of public information raises the level of information asymmetry in the government bond market. The results indicate that information asymmetry in the government bond market arises not from the absence of relevant public information, but rather the ability of market participants to interpret the information.

My work also contributes to the literature that studies endogenous network structure. There are other models that study the network formation in financial markets for different markets or from different perspectives, see [Bab16], [Zho14], and [CZ18].

The paper is organized as follows. The following section introduces the model setup of trading in the inter-dealer markets and the equilibrium concept of the trading game. In Section 1.3, I derive the Linear Bayesian Nash equilibrium of the trading game in opaque markets and show the information asymmetry decides dealers' surplus from asset reallocation. In Section 1.4, I introduce the public signal to the model; I study two different information structures of the public signal and show that the TRACE transparency would increase the degree of information asymmetry and lower dealers' trading surplus in markets with a relatively low degree of information asymmetry. In Section 1.5, I propose a network formation model to study the effect of TRACE on the inter-dealer network structure. In Section 1.6, I do empirical analysis to study the effect of TRACE post-trade transparency on inter-dealer trading activity in different markets. In Section 1.7, I study variants of the baseline model and show that the results of the baseline model are robust. Finally, I conclude. Proofs and data cleaning procedures are in the Appendix 4.1.

1.2 A Model of Trading in the Inter-dealer Markets

In this section I first describe the agents and the trading game they play, then I define the equilibrium of the trading game.

1.2.1 The Model Setup

1.2.1.1 Information Structure

We consider an economy with n risk-neutral dealers that trade bilaterally a divisible risky asset. The asset is in zero net supply.

Dealer i's value of the asset is

$$\theta_i = \theta + \eta_i,$$

where $\theta \sim \mathcal{N}(0, \sigma_\theta^2)$, $\eta_i \overset{i.i.d}{\sim} \mathcal{N}(0, \sigma_\eta^2)$, $\theta \perp \eta_i$. This implies that θ is normally distributed with a mean normalized to zero and a variance σ_θ^2, and η_i are drawn independently across dealers and from θ.

Each dealer i knows its own η_i, but does not know θ. Dealer 1 receives the private signal about θ,

$$s_1 = \theta + \varepsilon,$$

where $\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$, $\theta \perp \varepsilon$, $\eta_i \perp \varepsilon$.

In this model without public signal, other dealers do not receive any exogenous information. I define opaque markets to be the markets without public signal. I will introduce the public signal to the model in Section 1.4.

1.2.1.2 Dealer's Action and Payoff

Let $p_{i-1,i}$ or $p_{i,i-1}$ denote the price at which trade takes place over link $(i-1,i)$.

Suppose the network has one dealer, then there is no trade between dealers. Suppose the network has n dealers, then the trade takes place sequentially between dealer 1 and dealer 2, then between dealer 2 and dealer 3,...,between dealer $n-1$ and dealer n.

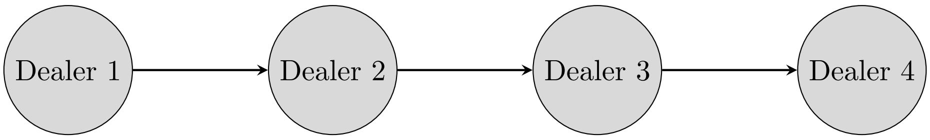

Figure 1.1: The Inter-dealer Network with $n = 4$

I assume there is demand from clients. The aggregate demand of clients on each link $(i, i+1)$ is $\beta p_{i,i+1}$, where $\beta < 0$. Clients' linear demand is micro-founded in the Appendix 4.1.2.[1]

The demand function of dealer 1 is

$$Q^1_{1,2}(\eta_1, s_1, p_{1,2}),$$

where $Q^1_{1,2}$ maps η_1, s_1, and $p_{1,2}$ into the quantity it wishes to trade on the link (1,2).

The demand functions of dealer $i \in \{2, .., n-1\}$ are

$$\left(Q^i_{i-1,i}(\eta_i, p_{i-1,i}), Q^i_{i,i+1}(\eta_i, p_{i-1,i}, p_{i,i+1})\right),$$

where $Q^i_{i-1,i}$ maps η_i, $p_{i-1,i}$ into the quantity it wishes to trade on the link $(i-1,i)$; and $Q^i_{i,i+1}$ maps η_i, $p_{i-1,i}$ and $p_{i,i+1}$ into the quantity it wishes to trade on the link $(i, i+1)$.

[1] I assume the clients are non-atomic and risk-neutral. They incur quadratic flow cost when trading. The quadratic flow cost is used in the models of [RW12] and [DZ17]. The fixed β for all the links is based on the assumption that clients only trade for liquidity needs. In Section 1.7.3, I relax this assumption and clients also trade for the asset payoff, and I show that the qualitative results in the baseline model still hold and the effect of transparency on dealers' trading surplus is amplified due to the effect of information asymmetry on clients' trading intensity.

The demand function of dealer n is

$$Q^n_{n-1,n}(\eta_n, p_{n-1,n}),$$

where $Q^n_{n-1,n}$ maps η_n, $p_{n-1,n}$ into the quantity it wishes to trade on the link $(n-1, n)$.

OTC trading protocols do not typically involve the submission of full demand schedules. But as mentioned in [BK18], generalized demand functions capture the repeated exchange of limit and market orders (i.e., the offer and acceptance of quotes) within a short time interval across fixed counterparties as a reduced-form price determination mechanism. [2]

The expected payoff of dealer 1 is

$$\mathbb{E}(\pi^1_{1,2}) = \mathbb{E}\big[Q^1_{1,2}(\eta_1, s_1, p_{1,2})(\theta_1 - p_{1,2})\big].$$

The expected payoff of dealer $i \in \{2, .., n-1\}$is

$$\mathbb{E}(\pi_i) = \mathbb{E}(\pi^i_{i-1,i}) + \mathbb{E}(\pi^i_{i,i+1}),$$

where

$$\mathbb{E}(\pi^i_{i-1,i}) = \mathbb{E}[Q^i_{i-1,i}(\eta_i, p_{i-1,i})(\theta_i - p_{i-1,i})],$$

$$\mathbb{E}(\pi^i_{i,i+1}) = \mathbb{E}[Q^i_{i,i+1}(\eta_i, p_{i-1,i}, p_{i,i+1})(\theta_i - p_{i,i+1})].$$

The expected payoff of dealer n is

$$\mathbb{E}(\pi^n_{n-1,n}) = \mathbb{E}\big[Q^n_{n-1,n}(\eta_n, p_{n-1,n})(\theta_n - p_{n-1,n})\big].$$

1.2.2 Equilibrium Concept

I consider linear equilibria of the game, defined as follows.

Definition 1. *A Linear Bayesian Nash equilibrium of the trading game is a vector of dealers' linear demand functions*

$$\Big\{Q^1_{1,2}(.), \big(Q^2_{1,2}(.), Q^2_{2,3}(.)\big), ..., \big(Q^{n-1}_{n-2,n-1}(.), Q^{n-1}_{n-1,n}(.)\big), Q^n_{n-1,n}(.)\Big\},$$

[2] In Section 1.7.2, I explicitly model the price-discovery process. The price discovery game shows that the equilibrium prices and quantities of my baseline model can be found via an iterative, decentralized process.

such that

(1) $Q^1_{1,2}(\eta_1, s_1, p_{1,2})$ *solves the problem*

$$\max_{Q^1_{1,2}} \mathbb{E}\big[Q^1_{1,2}(\theta_1 - p_{1,2})|\eta_1, s_1\big],$$

(2) $\big(Q^i_{i-1,i}(\eta_i, p_{i-1,i}), Q^i_{i,i+1}(\eta_i, p_{i-1,i}, p_{i,i+1})\big)$ *for* $i \in \{2, .., n-1\}$ *solves the problem*

$$\max_{(Q^i_{i-1,i}, Q^i_{i,i+1})} \mathbb{E}[Q^i_{i-1,i}(\theta_i - p_{i-1,i})|\eta_i] + \mathbb{E}[Q^i_{i,i+1}(\theta_i - p_{i,i+1})|\eta_i, p_{i-1,i}],$$

(3) $Q^n_{n-1,n}(\eta_n, p_{n-1,n})$ *solves the problem*

$$\max_{Q^n_{n-1,n}} \mathbb{E}\big[Q^n_{n-1,n}(\theta_n - p_{n-1,n})|\eta_n\big],$$

(4) $p_{1,2}$, $p_{i,i+1}$ *for* $i \in \{2, ..., n-1\}$ *clear the market,*

$$Q^1_{1,2}(\eta_1, s_1, p_{1,2}) + Q^2_{1,2}(\eta_2, p_{1,2}) + \beta p_{1,2} = 0,$$

$$Q^i_{i,i+1}(\eta_i, p_{i-1,i}, p_{i,i+1})) + Q^{i+1}_{i,i+1}(\eta_{i+1}, p_{i,i+1}) + \beta p_{i,i+1} = 0.$$

Each dealer chooses its demand functions for the transactions, to maximize its expected profits, given its private value (for dealer 1, also given its private information) and given the demand functions chosen by its counter-parties. For each transaction, given both dealers' demand functions, the equilibrium price clears the market. Implicit in the definition of the equilibrium is that each dealer understands that she has a price impact when trading with the counterparties.

1.3 Equilibrium of the Trading Game

In this section, I derive the equilibrium of the trading game in opaque markets. I show the information asymmetry between dealers decide how aggressively they trade on their private values in equilibrium, and thus decide the surplus from asset reallocation between them.

The following proposition characterizes the trading behavior of dealer 1 and dealer 2, and the price function on the link (1, 2).

Proposition 1. *In the Linear Bayesian Nash equilibrium of the trading game between dealer 1 and 2, dealer 1's demand function is*

$$Q^1_{1,2} = a^1_{1,2}s_1 + b^1_{1,2}p_{1,2} + c^1_{1,2}\eta_1.$$

Dealer 2's demand function on link (1,2) is

$$Q^2_{1,2} = b^2_{1,2}p_{1,2} + c^2_{1,2}\eta_2.$$

Define

$$\kappa_1 \equiv \frac{\sigma^2_\eta}{Var(\theta) - Var(\theta|s_1)} = (1 + \frac{\sigma^2_\varepsilon}{\sigma^2_\theta})\frac{\sigma^2_\eta}{\sigma^2_\theta} := (1+\gamma)\varphi,$$

which is the inverse of information asymmetry between dealer 1 and 2 scaled by σ^2_η. The degree of the interdependence between dealers' values is captured by $\varphi \equiv \frac{\sigma^2_\eta}{\sigma^2_\theta}$. The inverse of dealer 1's information precision is captured by $\gamma \equiv \frac{\sigma^2_\varepsilon}{\sigma^2_\theta}$.

$a^1_{1,2}, b^1_{1,2}, b^2_{1,2}, c^1_{1,2}, c^2_{1,2}$ *have closed-form solution*

$$\begin{aligned}
b^1_{1,2} &= b^2_{1,2} + \beta = \beta\kappa_1, \\
c^1_{1,2} &= c^2_{1,2} = -\beta\kappa_1, \\
\frac{c^1_{1,2}}{a^1_{1,2}} &= \frac{\kappa_1}{\varphi}.
\end{aligned}$$

The price function is

$$p_{1,2} = \frac{\varphi}{2\kappa_1}s_1 + \frac{\eta_1 + \eta_2}{2}.$$

In order to solve for the equilibrium of the trading game between dealer 2 and 3, we first analyze the information dealer 2 gets from trading with dealer 1. For dealer 2, the market clearing condition for link (1,2) implies

$$Q^2_{1,2} + a^1_{1,2}s_1 + b^1_{1,2}p_{1,2} + c^1_{1,2}\eta_1 + \beta p_{1,2} = 0,$$

thus

$$p_{1,2} = -\frac{a^1_{1,2}s_1 + c^1_{1,2}\eta_1}{b^1_{1,2} + \beta} - \frac{Q^2_{1,2}}{b^1_{1,2} + \beta} := I_2 + \lambda^2_{1,2}Q^2_{1,2}$$

Dealer 2 knows his or her own demand, so dealer 2 can infer the intercept I_2. Thus, dealer 2 can learn the private signal s_2,

$$s_2 \equiv s_1 + \frac{c^1_{1,2}}{a^1_{1,2}}\eta_1 = s_1 + \frac{\kappa_1}{\varphi}\eta_1.$$

Similarly, I define s_i ($i \in \{2, ..., n-1\}$) to be the signal dealer i learns from price $p_{i-1,i}$. Following the same equilibrium characterization procedure of Proposition 1, the trading game between dealer 2 and 3, ..., dealer $n-1$ and n can be solved, as shown in the following lemma.

Lemma 1. *In the Linear Bayesian Nash equilibrium of the trading game between dealer i and $i+1$ ($i \in \{2, 3, ..., n-1\}$), dealer i's demand function can be expressed as*

$$Q^i_{i,i+1} = a^i_{i,i+1} s_i + b^i_{i,i+1} p_{i,i+1} + c^i_{i,i+1} \eta_{i,i+1},$$

where

$$s_i = \frac{2\kappa_{i-1} p_{i-1,i} - \kappa_{i-1}\eta_i}{\varphi}.$$

Dealer $i+1$'s demand function on link $(i, i+1)$ is

$$Q^{i+1}_{i,i+1} = b^{i+1}_{i,i+1} p_{i,i+1} + c^{i+1}_{i,i+1} \eta_{i+1}.$$

Define

$$\kappa_i \equiv \frac{\sigma^2_\eta}{Var(\theta) - Var(\theta|s_i)},$$

which is the inverse of information asymmetry between dealer i and $i+1$ scaled by σ^2_η. $a^i_{i,i+1}, b^i_{i,i+1}, b^{i+1}_{i,i+1}, c^i_{i,i+1}, c^{i+1}_{i,i+1}$ have closed-form solution

$$b^i_{i,i+1} = b^{i+1}_{i,i+1} + \beta = \beta\kappa_i,$$
$$c^i_{i,i+1} = c^{i+1}_{i,i+1} = -\beta\kappa_i,$$
$$\frac{c^i_{i,i+1}}{a^i_{i,i+1}} = \frac{\kappa_i}{\varphi}.$$

The price function is

$$p_{i,i+1} = \frac{\varphi}{2\kappa_i} s_i + \frac{\eta_i + \eta_{i+1}}{2}.$$

In Proposition 1 and Lemma 1, from $c^i_{i,i+1} = c^{i+1}_{i,i+1} = -\beta\kappa_i$, we can see in equilibrium how aggressively dealers trade on their private values is decided by the inverse of information asymmetry between two consecutive dealers on each link scaled by the variance of private values, $\frac{\sigma^2_\eta}{Var(\theta) - Var(\theta|s_i)}$; it is also decided by clients' trading intensity. This will create important implications for dealers' trading surplus, which will be shown later.

The following lemma generalizes the private information diffusion process over links.

Lemma 2. *In equilibrium, dealer $i+1$ learns s_i from trading with dealer i,*

$$s_{i+1} = s_1 + \frac{\kappa_1}{\varphi}\eta_1 + \frac{\kappa_2}{\varphi}\eta_2 + ... + \frac{\kappa_i}{\varphi}\eta_i.$$

Define

$$\kappa_{i+1} \equiv \frac{\sigma_\eta^2}{Var(\theta) - Var(\theta|s_{i+1})} = \kappa_i + \kappa_i^2.$$

From Lemma 2, we can see the information diffusion process is governed by the initial information asymmetry between dealer 1 and 2. If the degree of informational asymmetry between dealer 1 and 2 is lower, then the degree of information asymmetry between all the subsequent consecutive dealers will become lower before their trade. This is because a lower degree of information asymmetry between dealer 1 and 2 will make a dealer trade relatively more aggressively on private value than private signal, reflected by the ratio $\frac{c_1}{a_1} = \frac{\kappa_1}{\varphi}$. Thus dealer 2 learns less from the price between dealer 1 and 2 and the degree of information asymmetry between dealer 2 and 3 is lower as well, which makes dealer 2 trade relatively more aggressively on private value than the signal learned from trading with dealer 1, and so on.

Now I characterize dealers' trading profit using the result from Proposition 1 and Lemma 1. Dealers' profit from each link can be decomposed into two components: the surplus from asset reallocation between dealers and the profits from serving clients. Considering dealer $i+1$ demands $Q_{i,i+1}^{i+1}$ units of the asset, clients demand $\beta p_{i,i+1}$ units of the asset, dealer i supplies $Q_{i,i+1}^{i+1} + \beta p_{i,i+1}$ units of the asset. The trading profit of dealer i and $i+1$ is

$$\mathbb{E}[(Q_{i,i+1}^{i+1}(\eta_{i+1} - \eta_i)] + \mathbb{E}[\beta p_{i,i+1}(p_{i,i+1} - \theta - \eta_i)],$$

where the first component is the surplus from the asset reallocation between dealers, and the second component is the rent dealer i extracts from serving the clients.

The following lemma shows dealers' profit on each link.

Lemma 3. *In equilibrium, the ex-ante trading profit of dealer i and $i+1$ from link $(i, i+1)$ is*

$$\mathbb{E}(\pi_{i,i+1}^i) + \mathbb{E}(\pi_{i,i+1}^{i+1}) = -\beta\sigma_\eta^2\kappa_i + \frac{-\beta}{4}\frac{\sigma_\eta^2}{\kappa_i},$$

where $-\beta\sigma_\eta^2\kappa_i$ is the surplus from asset reallocation between dealer i and $i+1$, $\frac{-\beta}{4}\frac{\sigma_\eta^2}{\kappa_i}$ is dealer i's profit from serving clients.

Lemma 3 shows less severe adverse selection increases the surplus from asset reallocation between dealers. This is because the lower degree of information asymmetry will make dealers trade more aggressively on their private values, and thus increase the trading surplus from asset reallocation between them.

To link my theory to more observable characteristics of the inter-dealer markets documented in the empirical literature, I define the inter-dealer trading cost of dealer $i \in \{2, ..., n-1\}$ to be $|p_{i,i+1} - p_{i-1,i}|$. This definition of trading cost is similar to that in [LS19], which measures trading costs for investors by inter-dealer markups.

Lemma 4. *For* $i \in \{2, ..., n-1\}$,

$$Var(p_{i,i+1} - p_{i-1,i}) = \frac{\sigma_\eta^2}{4} \frac{\kappa_{i-1}}{\kappa_{i-1} + 1} + \frac{1}{4}\sigma_\eta^2,$$

thus

(1) Inter-dealer trading cost $\mathbb{E}(|p_{i,i+1} - p_{i-1,i}|) = \sqrt{\frac{2}{\pi}}\sqrt{Var(p_{i,i+1} - p_{i-1,i})}$ *is increasing in* i.

(2) The average inter-dealer trading cost $\frac{\sum_{i \in \{2,...,n-1\}} \mathbb{E}(|p_{i,i+1} - p_{i-1,i}|)}{n-2}$ *is increasing in* n.

Lemma 4 shows that the inter-dealer trading cost is increasing over links; thus, the average inter-dealer trading cost of the whole chain is increasing in number of dealers in the chain. This result is consistent with the empirical finding in [LS19] that average markups increase monotonically with the number of dealers intermediating the chain, and the evidence in [Sch12] on newly issued bonds, from 1.9% on average when one dealer is involved to 3.7% with seven dealers involved.

1.4 Exogenous and Endogenous Public Signal

In this Section, I first introduce the public signal to the trading game, and derive the equilibrium of the trading game. Using the equilibrium solution, I study in both cases how the information diffuses and how it affects dealers' trading surplus from asset reallocation between them. Then I study two different information structures of the public signal and highlight the different implications in these two scenarios.

1.4.1 Introduction of a Public Signal

For simplicity, I consider the release of just one public signal S. To allow delays in the information release, as is done in practice, I consider that the signal is made available to all traders after the trade between dealer $n_p - 1$ and n_p is done.

$$S = \theta + \varepsilon_S,$$

where $\varepsilon_S \sim \mathcal{N}(0, \sigma_p^2)$, $\theta \perp \varepsilon_S$, $\varepsilon_S \perp \eta_{n_p}, \eta_{n_p+1}, ..., \eta_n$. This implies that ε_S is normally distributed with a variance σ_S and is independent from θ. I assume the noisy term in the public signal is orthogonal to the private values of dealer $n_p, n_p+1, ..., n$. We consider the natural case that these dealers' private values do not affect the generation of the public signal.

With the existence of the public signal S, for dealer $i \in \{n_p, ..., n\}$, their demand depends on this public signal. The demand functions of $i \in \{n_p, .., n-1\}$ are

$$\big(Q^i_{i-1,i}(\eta_i, p_{i-1,i}, S), Q^i_{i,i+1}(\eta_i, p_{i-1,i}, p_{i,i+1}, S)\big).$$

The demand function of dealer n is

$$Q^n_{n-1,n}(\eta_n, p_{n-1,n}, S).$$

Let $\hat{s}_i$ denote the private signal dealer i learns from trading with dealer $i-1$. From Lemma 2, we have the result that for $i \in \{2, ..., n_p\}$,

$$\hat{s}_i = s_i = s_1 + \frac{\kappa_1}{\varphi}\eta_1 + \frac{\kappa_2}{\varphi}\eta_2 + ... + \frac{\kappa_{i-1}}{\varphi}\eta_{i-1},$$

as their trade takes place before the dissemination of the public signal, and the information diffusion until dealer n_p is not affected. By contrast, for dealer $i \in \{n_p + 1, ..., n-1\}$, $\hat{s}_i \neq s_i$ as the public signal changes the information diffusion process since the trade between dealer n_p and $n_p + 1$.

The following proposition characterizes the trading behavior of dealer i and $i+1$ for $i \in \{n_p, n_p + 1, ..., n-1\}$, and the price function on the link $(i, i+1)$.

Proposition 2. *In the Linear Bayesian Nash equilibrium of the trading game between dealer i and $i+1$ ($i \in \{n_p, n_p+1, ..., n-1\}$), dealer i's demand is*

$$Q^i_{i,i+1} = a^i_{i,i+1}\hat{s}_i + b^i_{i,i+1}p_{i,i+1} + c^i_{i,i+1}\eta_i + d^i_{i,i+1}S,$$

dealer $i+1$'s demand is

$$Q^{i+1}_{i,i+1} = b^{i+1}_{i,i+1} p_{i,i+1} + c^{i+1}_{i,i+1} \eta_{i+1} + d^{i+1}_{i,i+1} S.$$

Define

$$\hat{\kappa}_i \equiv \frac{\sigma^2_\eta}{Var(\theta|S) - Var(\theta|\hat{s}_i, S)},$$

which is the inverse of information asymmetry between dealer i and $i+1$ scaled by σ^2_η. The coefficients have closed-form solution

$$b^i_{i,i+1} = b^{i+1}_{i,i+1} + \beta = \beta\hat{\kappa}_i, \qquad c^i_{i,i+1} = c^{i+1}_{i,i+1} = -\beta\hat{\kappa}_i,$$
$$\frac{c^i_{i,i+1}}{a^i_{i,i+1}} = \frac{\hat{\kappa}_i}{\varphi}\left(1 - \frac{Cov(\hat{s}_i, S)}{Var(S)}\right).$$

The price function is

$$p_{i,i+1} = -\frac{a^i_{i,i+1}}{2\hat{\kappa}_i \beta}\hat{s}_i - \frac{d^i_{i,i+1} + d^{i+1}_{i,i+1}}{2\hat{\kappa}_i\beta} S + \frac{\eta_i + \eta_{i+1}}{2}.$$

Proposition 2 shows that the equilibrium of the trading game with a public signal is very similar to the equilibrium without, but with the following two differences. First, dealers' demands and the price function in equilibrium depend on the public signal. Second, the public signal affects the information asymmetry between dealers. Third, the private information diffusion process is not only decided by the information asymmetry $\hat{\kappa}_i$, but also by a new term $\frac{Cov(\hat{s}_i,S)}{Var(S)}$.

For dealer n_p and n_p+1, their information asymmetry in the game with public signal, $Var(\theta|S) - Var(\theta|s_{n_p}, S)$, is different from that in the game without public signal, $Var(\theta) - Var(\theta|s_{n_p})$, as both of them learn from the public signal. Dealer n_p could learn a greater or smaller extent from S, $Var(\theta|s_{n_p}) - Var(\theta|s_{n_p}, S)$, than dealer n_p+1, $Var(\theta) - Var(\theta|S)$.

For dealer i and $i+1$ ($i \in \{n_p+1, ..., n-1\}$), their information asymmetry in the game with public signal, $Var(\theta|S) - Var(\theta|\hat{s}_i, S)$, is different from that in the game without public signal, $Var(\theta) - Var(\theta|\hat{s}_i)$, as both of them learn from the public signal, and also the private signals learned by dealer i, $\hat{s}_i$ and s_i, are different in these two cases. Dealer i could learn a greater or smaller extent, $Var(\theta|s_i) - Var(\theta|\hat{s}_i, S)$, than dealer $i+1$, $Var(\theta) - Var(\theta|S)$.

The following lemma shows the information diffusion process after the release of the public information.

Lemma 5. *In equilibrium, dealer $i+1$ learns $\hat{s}_i$ from trading with dealer i ($i \in \{n_p, n_p + 1, ..., n-1\}$),*

$$s_{i+1} = s_{n_p} + \frac{\hat{\kappa}_{n_p}}{\varphi}(1 - \frac{Cov(\hat{s}_{n_p}, S)}{Var(S)})\eta_{n_p} + ... + \frac{\hat{\kappa}_i}{\varphi}(1 - \frac{Cov(\hat{s}_i, S)}{Var(S)})\eta_i,$$
$$\hat{\kappa}_{i+1} = \hat{\kappa}_i + \hat{\kappa}_i^2.$$

Using Lemma 2 and Lemma 5, we immediately have the following corollary that characterizes the necessary and sufficient condition for the effect of the public signal on the information asymmetry between dealer i and $i+1$ ($i \in \{n_p, n_p+1, ..., n-1\}$).

Corollary 1. *Comparing the effect of the public signal on the information asymmetry between dealer i and $i+1$ for $i \in \{n_p + 1, ..., n-1\}$,*

$$Var(\theta|S) - Var(\theta|\hat{s}_i, S) > Var(\theta) - Var(\theta|s_i)$$

$$\textit{iff} \quad Var(\theta|S) - Var(\theta|s_{n_p}, S) > Var(\theta) - Var(\theta|s_{n_p}).$$

If and only if the degree of information asymmetry between dealer n_p and $n_p + 1$ becomes higher due to the existence of the public signal, then for any dealer $i \in \{n_p+1, ..., n-1\}$, the degree of information asymmetry between dealer i and $i+1$ becomes higher.

Corollary 1 shows that, to determine whether the introduction of a public signal mitigates or worsens adverse selection, it is sufficient to focus on the link immediately following the information release.

Now using the results in Proposition 2, I characterize dealers' surplus from the asset reallocation, as shown in the following lemma.

Lemma 6. *In equilibrium, the surplus from asset reallocation between dealer i and $i+1$ ($i \in \{n_p, n_p + 1, ..., n-1\}$) is*

$$\mathbb{E}[(Q_{i,i+1}^{i+1}(\eta_{i+1} - \eta_i)] = -\beta\sigma_\eta^2\hat{\kappa}_i = \frac{-\beta\sigma_\eta^4}{Var(\theta|S) - Var(\theta|\hat{s}_i, S)}.$$

Compared with its counterpart in the game without public signal

$$\mathbb{E}[(Q_{i,i+1}^{i+1}(\eta_{i+1}-\eta_i)] = \frac{-\beta\sigma_\eta^4}{Var(\theta) - Var(\theta|s_i)},$$

we can see whether the existence of a public signal would increase or reduce the surplus from asset reallocation between dealers depends on whether the degree of information asymmetry between dealers becomes higher or lower. Equivalently, it depends on whether or not dealer n_p can learn more from the public signal than dealer n_p+1.

Now I study two different information structures of the public signal. I first show the effect of transparency on dealers' trading surplus when the public signal is exogenous. Then I show the effect of transparency on dealers' trading surplus after endogenizing the public signal as the disseminated price of TRACE. I highlight the different implications in these two scenarios.

1.4.2 Exogenous Public Signal

I assume the public signal is exogenous, in the sense that the noisy term in S is independent from the noisy term in s_{n_p},

$$\varepsilon_S \perp (\varepsilon + \frac{\kappa_1}{\varphi}\eta_1 + \frac{\kappa_2}{\varphi}\eta_2 + ... + \frac{\kappa_{n_p-1}}{\varphi}\eta_{n_p-1}).$$

It can be shown that dealer n_p+1 will learn more from the exogenous public signal than dealer n_p. Using Corollary 1, we have the result that the degree of information asymmetry between dealer i and $i+1$ becomes lower for $i \in \{n_p, n_p+1, ..., n-1\}$, as shown in the following lemma.

Lemma 7. *If the public signal is exogenous, then in equilibrium, the degree of information asymmetry between dealer i and $i+1$ ($i \in \{n_p, n_p+1, ..., n-1\}$) is lower than the case without a public signal,*

$$Var(\theta|S) - Var(\theta|\hat{s}_i, S) < Var(\theta) - Var(\theta|s_i).$$

We can also interpret this result from the perspective of information substitution and complementarity, e.g.[BHK13]. If the public signal is exogenous,

$$Var(\theta|S) - Var(\theta|s_{n_p}, S) < Var(\theta) - Var(\theta|s_{n_p}),$$

thus in terms of reducing the posterior variance of θ, the marginal value of having s_{n_p} is smaller with the existence of S than without the existence of S. Thus the public signal S is always a substitute for the private signal s_{n_p}.

The existence of the exogenous public signal would affect dealers' profits from clients, while it can be shown that the extent to which that is affected is modest. In contrast, the extent to which dealers' surplus from asset reallocation is affected is dramatic if the degree of information asymmetry becomes relatively low.

Lemma 8.

$$\lim_{\sigma_p \to 0} \mathbb{E}[(Q_{i,i+1}^{i+1}(\eta_{i+1} - \eta_i)] = \lim_{\sigma_p \to 0} \frac{-\beta \sigma_\eta^4}{Var(\theta|S) - Var(\theta|\hat{s}_i, S)} = \infty,$$

$$\lim_{\sigma_p \to 0} \mathbb{E}[\beta p_{i,i+1}(p_{i,i+1} - \theta - \eta_i)] \quad \textit{is bounded.}$$

If the public signal is extremely precise, the information asymmetry goes away, and the surplus from asset reallocation between dealer $i, i+1$ will explode, while dealers' profit from serving clients is bounded. Thus the profit of dealer i and $i+1$ from link $(i, i+1)$ is larger with a precise public signal than without a public signal.

This result is consistent with the conventional wisdom. The existence of the public signal reduces the information asymmetry, makes the adverse selection less severe, and thus increases the trading surplus in equilibrium by facilitating dealers' trade.

1.4.3 Endogenous Public Signal

In practice, TRACE collects the price of trades that have taken place and then disseminates the historical prices to the public. Thus the disseminated prices from the upstream trades are not exogenous public signals, because firstly, the prices of the upstream trades and the prices of the downstream trades could share the same information source (in my model, dealer 1's private signal s_1); secondly, they share the noisy terms that arise from upstream dealers' idiosyncratic trading needs (in my model, for example, dealer 1's private value η_1 enters the noisy terms of the prices in all the links.)

I assume the price of the first trade, the trading price between dealer 1 and 2, is the public signal,

$$S = \frac{2\kappa_1}{\varphi} p_{1,2} = s_1 + \frac{\kappa_1}{\varphi}(\eta_1 + \eta_2).$$

I make a technical assumption that $n_p \geq 3$. Namely, $p_{1,2}$ is observable after the trade between dealer 2 and 3 has finished. This assumption ensures that dealer 2 does not have an incentive to manipulate the public signal $p_{1,2}$ to profit from the trade with dealer 3.[3]

The following result shows that in markets with a relatively low degree of information asymmetry, the public signal would increase the degree of information asymmetry.

Proposition 3. *Compare the information asymmetry between dealer i and $i+1$ ($i \in \{n_p, ..., n-1\}$) with and without public signal, there is a threshold κ^* such that,*

$$Var(\theta|S) - Var(\theta|\hat{s}_i, S) > Var(\theta) - Var(\theta|s_i) \quad \textit{if } \kappa_1 > \kappa^*.$$

If $n_p = 3$, $\kappa^* = 1$ is necessary and sufficient.

From the perspective of information substitution and complementarity, e.g.[BHK13], we can see when $\kappa_1 > \kappa^*$,

$$Var(\theta|S) - Var(\theta|s_{n_p}, S) > Var(\theta) - Var(\theta|s_{n_p}),$$

thus in terms of reducing the posterior variance of θ, the marginal value of having s_{n_p} is larger with the existence of S than without the existence of S. Thus the public signal S is a complement for the private signal s_{n_p}.

As shown in Corollary 1, for dealer i and $i+1$ ($i \in \{n_p, n_p+1, ..., n-1\}$), whether their information asymmetry in the game with a public signal, $Var(\theta|S) - Var(\theta|\hat{s}_i, S)$, is larger or smaller that in the game without a public signal, $Var(\theta) - Var(\theta|s_i)$, depends on whether the information asymmetry between dealer n_p and n_p+1, $Var(\theta|S) - Var(\theta|\hat{s}_{n_p}, S)$, is larger or smaller that in the game without a public signal, $Var(\theta) - Var(\theta|s_{n_p})$. That is equivalent to whether dealer n_p learns a greater or smaller extent from S, $Var(\theta|s_{n_p}) - Var(\theta|s_{n_p}, S)$, than dealer n_p+1, $Var(\theta) - Var(\theta|S)$.

[3]It can be proved that if we compare the cases that $p_{1,2}$ is observable or not for dealer 3 before dealer 3 trades with dealer 2, dealer 2 will behave differently in equilibrium. It is an interesting question how the price transparency affects price formation and information diffusion by incentivizing dealers to manipulate prices, though it is beyond the scope of this paper.

Figure 1.2 plots the inverse of variance reduction for dealer 3 and 4 due to the existence of the public signal when $n_p = 3$.

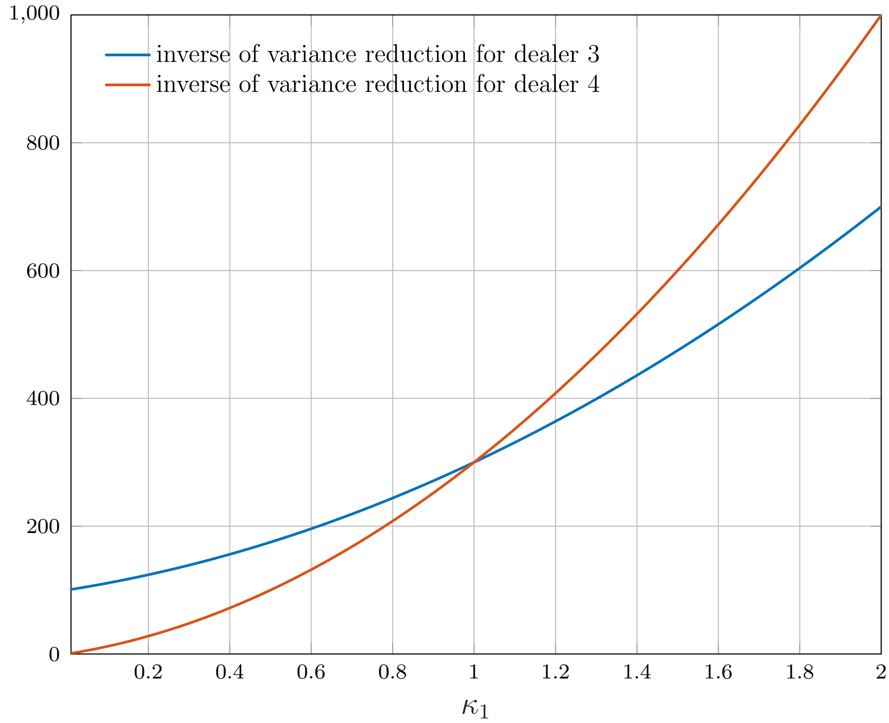

Figure 1.2: The Inverse of Variance Reduction

$\frac{1}{Var(\theta|s_3)-Var(\theta|s_3,S)}$ and $\frac{1}{Var(\theta)-Var(\theta|S)}$

Parameters: $n_p = 3, \sigma_\theta = 1, \sigma_\eta = 0.1$.

To see the intuition for this result, let us look at the case $n_p = 3$. In equilibrium, dealer 3's private signal is $s_3 = s_1 + \frac{\kappa_1}{\varphi}\eta_1 + \frac{\kappa_2}{\varphi}\eta_2$, and the public signal is $S = s_1 + \frac{\kappa_1}{\varphi}(\eta_1 + \eta_2)$. The key difference between the endogenous and the exogenous signal is that the endogenous signal is both about the asset payoff and previous dealers' private values, which allows the more informed dealer to filter its private signal more effectively. Intuitively, dealer $n_p = 3$ already has a signal about the private values of dealer 1 and 2. The endogenous public signal provides an additional signal that can make dealer 3 fully informed about η_2 and, by implication, fully informed about $s_2 = s_1 + \frac{\kappa_1}{\varphi}\eta_1$. But dealer 4 only learns S. When k_1 is relatively large, $\frac{\kappa_1}{\varphi}\eta_2$, the difference between s_2 and S that stems from the private value of dealer 2, becomes more noisy, and would increase the degree of information asymmetry between dealer 3 and 4.

To further see this, let us do a thought experiment by assuming the public signal is

$s_1 + \frac{\kappa_1}{\varphi}\eta_1 + \frac{\kappa^*}{\varphi}\eta_2$. Dealer 3's private signal is the same as before. Then from this public signal and $s_3 = s_1 + \frac{\kappa_1}{\varphi}\eta_1 + \frac{\kappa_2}{\varphi}\eta_2$, dealer 3 still learns the signal $s_2 = s_1 + \frac{\kappa_1}{\varphi}\eta_1$, while dealer 4 learns the signal $s_1 + \frac{\kappa_1}{\varphi}\eta_1 + \frac{\kappa^*}{\varphi}\eta_2$. We can see the value of κ^* that governs how pronounced the private value of dealer 2 is in the public signal, would decide whether the public signal would increase or reduce the degree of information asymmetry between dealer 3 and 4. Considering κ^* approaches 0, then the information asymmetry between dealer 3 and 4 is almost removed; thus, the degree of information asymmetry becomes smaller. Suppose κ^* goes to infinity, then dealer 4 basically can not learn anything from the public signal (the information asymmetry between 3 and 4 approaches the information asymmetry between dealer 2 and 3); thus, the degree of information asymmetry becomes larger. [4]

From this thought experiment, we can see the effect of the public signal on the degree of information asymmetry is decided by the magnitude of κ^* or how significantly dealer 2's private value affects the public signal. From the price function shown in Proposition 1, we have in equilibrium $\kappa^* = \kappa_1$. The degree of information asymmetry between dealer 1 and 2 decides how aggressively dealer 2 trades on its private value, and thus decides how significantly its private value affects the public signal, which decides whether the public signal would increase or reduce the degree of information asymmetry between the subsequent dealers.

The existence of the endogenous public signal would affect dealers' profits from serving the clients, while it can be shown that qualitatively, that effect is dominated by the effect on dealers' surplus from asset reallocation if the degree of information asymmetry is relatively low, as shown by the following proposition.

Proposition 4. *Trading profit of dealer i and $i+1$ ($i \in \{n_p, ..., n-1\}$)*

$$\mathbb{E}(\pi^i_{i,i+1}) + \mathbb{E}(\pi^{i+1}_{i,i+1}) = \mathbb{E}[(Q^{i+1}_{i,i+1}(\eta_{i+1} - \eta_i)] + \mathbb{E}[\beta p_{i,i+1}(p_{i,i+1} - \theta - \eta_i)],$$

[4] Formally, the change of information asymmetry is

$$Var(\theta|s_1 + \frac{\kappa_1}{\varphi}\eta_1 + \frac{\kappa^*}{\varphi}\eta_2) - Var(\theta|s_3, s_1 + \frac{\kappa_1}{\varphi}\eta_1 + \frac{\kappa^*}{\varphi}\eta_2) - (Var(\theta) - Var(\theta|s_3))$$

$$= -\frac{\sigma^2_\theta}{\kappa_1 + \kappa_1^2 + \kappa_1^{*2}}\varphi + \frac{\sigma^2_\theta}{\kappa_2}\varphi - \frac{\sigma^2_\theta}{\kappa_3}\varphi = \sigma^2_\theta\varphi(\frac{1}{\kappa_2} - \frac{1}{\kappa_3} - \frac{1}{\kappa_1 + \kappa_1^2 + \kappa_1^{*2}}) = \sigma^2_\theta\varphi\frac{\kappa_2({\kappa^*}^2 - 1)}{(\kappa_2 + \kappa_2^2)(\kappa_1 + \kappa_1^2 + {\kappa^*}^2)}.$$

is smaller with a public signal than without a public signal if κ_1 is relatively large.

Figure 1.3 plots when $n_p = 3$, dealer 3 and dealer 4's trading profits from link (3,4), and dealer 4 and dealer 5's trading profits from link (4,5). The blue line plots the profits when the market is opaque, and the orange line plots the profits when the trading price between dealer 1 and 2, $p_{1,2}$, is public.

From the simulation results, we can see when κ_1 is above a threshold that is very close to 1 — which is a threshold for the information asymmetry to be larger or smaller — dealers' trading profits from link (3,4) or link (4,5) are higher in the case that the market is opaque. The observation from Figure 1.3 that these two thresholds are almost the same holds for all the parameters that I have used for the simulation and for any $n_p \geq 3$. That indicates that for the wide range of κ_1, the effect on dealers' information asymmetry, and thus on dealers' trading surplus from asset reallocation, plays a dominant role in affecting dealers' trading profit.[5]

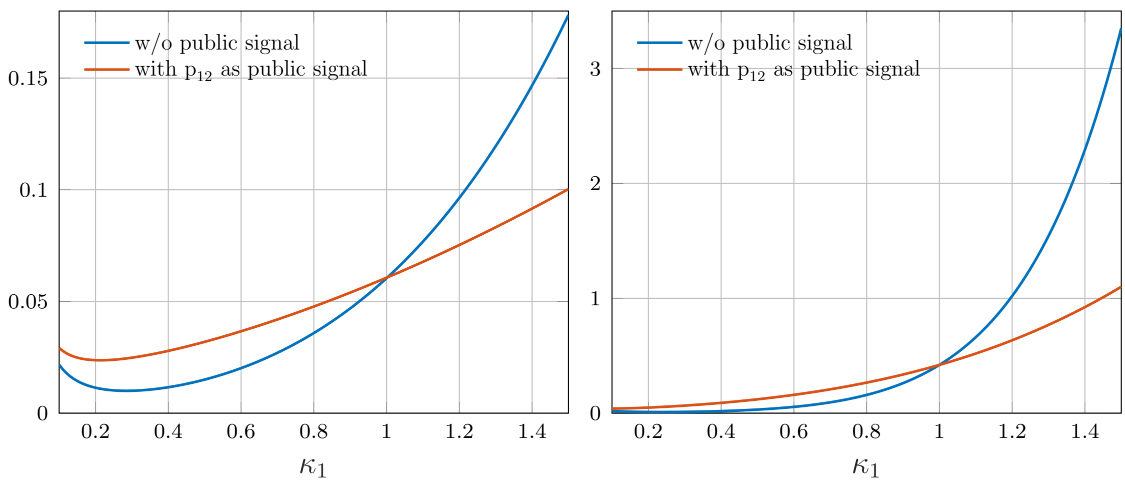

Figure 1.3: Profits on Links

$\mathbb{E}(\pi^3_{3,4}) + \mathbb{E}(\pi^4_{3,4})$, $\mathbb{E}(\pi^4_{4,5}) + \mathbb{E}(\pi^5_{4,5})$

Parameters: $n_p = 3$, $\beta = -1$, $\sigma_\theta = 1, \sigma_\eta = 0.1$.

[5]When κ_1 is very close to 0, sometimes dealers' trading profit is higher in the case that the market is opaque.

1.5 The Network Formation Game

In this section, I propose a network formation model to study the effect of TRACE on the inter-dealer network structure. The network formation stage is before the trading stage.

In this model, the dealer with private information about the asset payoff bargains with the second dealer over how to split the link formation cost; the second dealer who forms the link bargains with the third dealer over how to split the link formation cost, etc. In equilibrium, the surplus of each formed link would spread over all the dealers before this link and affect their network formation decision. Thus the effect of TRACE on downstream dealers' trading profit would affect the upstream dealers' link formation decision. The lower trading profits of downstream dealers could hurt the the formation of the whole network.

I show that, in a market with less information asymmetry, the inter-dealer trading frequency becomes lower after TRACE. The network formation game also implies that TRACE could reduce the trading frequency of the upstream dealers as well, because they benefit less from the downstream dealers' trading profits if forming the links. The empirical implications are tested in Section 1.6 and leave to be done more empirical work in the future.

1.5.1 The Model Setup

There is a set of risk-neutral dealers $\{1, 2, .., \bar{n}\}$. The link formation cost C is drawn from distribution $F(C)$. Given the realization of C, the link formation problem between dealer 1 and 2 is modeled as Nash's two-person bargaining problem with fixed disagreement payoffs, which are 0; if dealer 1 and 2 agree to form the link, we proceed to the link formation problem between dealer 2 and 3, etc; between two consecutive dealers, whether the agreement is reached and how the cost C is split are determined according to the symmetric Nash bargaining solution. For $i \in \{2, .., \bar{n}\}$, $l_{i-1,i} = 1$ denotes the link is formed between dealer $i-1$ and dealer i; $l_{i-1,i} = 0$ otherwise. $(i-1, i)$ denotes the formed link between dealer $i-1$ and dealer i.

$C^{i-1}_{i-1,i}$ and $C^{i}_{i-1,i}$ denote the costs paid by dealer $i-1$ and dealer i respectively when the link is formed between them. $C^{i-1}_{i-1,i} + C^{i}_{i-1,i} = C$. $\mathbb{E}(\pi^{i}_{i-1,i})$ and $\mathbb{E}(\pi^{i}_{i,i+1})$ denotes the expected payoff of dealer i from trading on the link $(i-1, i)$ and link $(i, i+1)$ respectively.

Dealer 1's payoff

$$U_1(l_{1,2}, C^1_{1,2}) = l_{1,2}(\mathbb{E}(\pi^1_{1,2}) - C^1_{1,2}).$$

Dealer 2's payoff

$$U_2(l_{1,2}, C^2_{1,2}, l_{2,3}, C^2_{2,3}) = l_{1,2}\big(\mathbb{E}(\pi^2_{1,2}) - C^2_{1,2} + l_{2,3}(\mathbb{E}(\pi^2_{2,3}) - C^2_{2,3})\big).$$

For $i \in \{2, ..., \bar{n} - 1\}$, dealer i's payoff is

$$U_i(l_{i-1,i}, C^i_{i-1,i}, l_{i,i+1}, C^i_{i,i+1}) = l_{i-1,i}\big(\mathbb{E}(\pi^i_{i-1,i}) - C^i_{i-1,i} + l_{i,i+1}(\mathbb{E}(\pi^i_{i,i+1}) - C^i_{i,i+1})\big).$$

For dealer $\bar{n}$, its payoff is

$$U_{\bar{n}}(l_{\bar{n}-1,\bar{n}}, C^{\bar{n}}_{\bar{n}-1,\bar{n}}) = l_{\bar{n}-1,\bar{n}}\big(\mathbb{E}(\pi^{\bar{n}}_{\bar{n}-1,\bar{n}}) - C^{\bar{n}}_{\bar{n}-1,\bar{n}}\big).$$

Whether the link is formed and how the cost is split are determined according to the symmetric Nash bargaining solution.

In the main text of the paper, I restrict the network to a line or chain. This framework can be extended to study the formation of any tree network, but all the qualitative results will be the same as what I show in the main text.[6] After the network is formed, dealers' private values and dealer 1's private signal s_1 are realized, and then the sequential trading takes place.[7]

1.5.2 Equilibrium Concept

Definition 2. *n denotes the number of dealers in network. $\big(n, (\hat{l}_{i-1,i})_{i\in\{2,..,n\}}, (\hat{C}^{i-1}_{i-1,i})_{i\in\{2,..,n\}}\big)$ is an equilibrium if and only if*

[6]For example, I am able to extend the network formation game, such that the dealer that has formed the link can contact and negotiate with each of a random number of other dealers, and decide whether to form links with them. The details are available upon request.

[7]This setup of timing is in the spirit of security design literature, for example, [DD99], such that I do not need to deal with the thorny signaling problem during the network formation process. The network structure in equilibrium only depends on the primitives of the model, rather than the realization of the random variables.

(1) $\hat{C}^{i-1}_{i-1,i}$ is the symmetric Nash bargaining solution

$$\hat{C}^{i-1}_{i-1,i} = \arg\max_{C^{i-1}_{i-1,i}} \left(\mathbb{E}(\pi^{i-1}_{i-1,i}) - C^{i-1}_{i-1,i}\right)^{\frac{1}{2}},$$

$$\left(\mathbb{E}(\pi^{i}_{i-1,i}) - (C - C^{i-1}_{i-1,i}) + 1(i < n)\big((\mathbb{E}(\pi^{i}_{i,i+1}) - \hat{C}^{i}_{i,i+1})\right)^{\frac{1}{2}},$$

where $\big((\mathbb{E}(\pi^{i}_{i,i+1}) - \hat{C}^{i}_{i,i+1}\big)$ *is the continuation value for* $i < n$*. Thus* $\hat{l}_{i-1,i} = 1$ *if and only if* $\mathbb{E}(\pi^{i-1}_{i-1,i}) \geq C^{i-1}_{i-1,i}$ *and* $\hat{l}_{j-1,j} = 1$ *for all* $j \in \{2, ..., i-1\}$*.*

(2) Let $\hat{l}_{0,1} = 1$*.* n *is the maximal integer in* $\{1, ..., \bar{n}\}$ *such that* $\hat{l}_{i-1,i} = 1$ *for any* $i \in \{1, ..., n\}$*.*

1.5.3 Equilibrium of the Network Formation Game

Conditional on all the previous links being formed, whether the last link can be formed or not just depends on whether the surplus of trading on this link, $\mathbb{E}(\pi^{n-1}_{n-1,n}) + \mathbb{E}(\pi^{n}_{n-1,n})$, is larger than the link formation cost.

Conditional on all the links before the penultimate link being formed, $\mathbb{E}(\pi^{i-1}_{i-1,i}) + \mathbb{E}(\pi^{i}_{i-1,i}) - C \geq 0$ does not ensure the penultimate link can be formed. For the penultimate link in the chain, whether it can be formed or not depends on the surplus of trading on this link and the net benefit of dealer $n-1$ from the last link, $\mathbb{E}(\pi^{i}_{i,i+1}) - \hat{C}^{i}_{i,i+1}$, which is affected by the surplus of the last link. Solving the symmetric Nash Bargaining problem for the penultimate link and substituting in the symmetric Nash bargaining solution of the last link, we have the penultimate link able to be formed if and only if

$$\frac{\mathbb{E}(\pi^{n-2}_{n-2,n-1}) + \mathbb{E}(\pi^{n-1}_{n-2,n-1}) + \frac{1}{2}\left(\mathbb{E}(\pi^{n-1}_{n-1,n} + \mathbb{E}(\pi^{n}_{n-1,n})\right)}{1 + \frac{1}{2}} \geq C.$$

Repeating this procedure for all the previous links until the first link, I can get the cost threshold for each length of the network. If the realized cost is below that threshold, the network with that length can be formed in equilibrium. The following algorithm formalizes this procedure.

Algorithm that solves the network formation game

Considering $n = \bar{n}$.

Link $(\bar{n}-1, \bar{n})$ is formed if and only if $C \leq \tilde{C}_{\bar{n}-1,\bar{n}}(\bar{n})$, where

$$\tilde{C}_{\bar{n}-1,\bar{n}}(\bar{n}) \equiv \mathbb{E}(\pi^{\bar{n}-1}_{\bar{n}-1,\bar{n}}) + \mathbb{E}(\pi^{\bar{n}}_{\bar{n}-1,\bar{n}}).$$

Link $(i-1, i)$ for $i \in \{2, \bar{n}\}$ is formed if and only if $C \leq \tilde{C}_{i-1,i}(\bar{n})$, where

$$\tilde{C}_{i-1,i}(\bar{n}) \equiv \frac{\mathbb{E}(\pi^{i-1}_{i-1,i}) + \mathbb{E}(\pi^{i}_{i-1,i}) + \frac{1}{2}\left(\mathbb{E}(\pi^{i}_{i,i+1}) + \mathbb{E}(\pi^{i+1}_{i,i+1})\right) + \ldots \frac{1}{2^{\bar{n}-i}}\left(\mathbb{E}(\pi^{\bar{n}-1}_{\bar{n}-1,\bar{n}}) + \mathbb{E}(\pi^{\bar{n}}_{\bar{n}-1,\bar{n}})\right)}{1 + \frac{1}{2} + \ldots \frac{1}{2^{\bar{n}-i}}}.$$

Define $C^*(\bar{n}) \equiv \min\{\tilde{C}_{1,2}(\bar{n}), \ldots, \tilde{C}_{\bar{n}-1,\bar{n}}(\bar{n})\}$.

In equilibrium the trading network has $\bar{n}$ dealers if and only if $C \leq C^*(\bar{n})$.

If $C > C^*(\bar{n})$, we let $n = \bar{n}-1$, and characterize $C^*(\bar{n}-1) \equiv \min\{\tilde{C}_{1,2}(\bar{n}-1), \ldots, \tilde{C}_{\bar{n}-2,\bar{n}-1}(\bar{n}-1)\}$.

In equilibrium the network has $\bar{n}-1$ dealers if and only if $C^*(\bar{n}) < C \leq C^*(\bar{n}-1)$.

If $C > \max\{C^*(\bar{n}), C^*(\bar{n}-1)\}$, we let $n = \bar{n}-2$, and characterize $C^*(\bar{n}-2) \equiv \min\{\tilde{C}_{1,2}(\bar{n}-2), \ldots, \tilde{C}_{\bar{n}-3,\bar{n}-2}(\bar{n}-2)\}$.

In equilibrium the network has $\bar{n}-2$ dealers if and only if $\max\{C^*(\bar{n}), C^*(\bar{n}-1)\} < C \leq C^*(\bar{n}-2)$.

...

The link between dealer $i-1$ and i is formed in equilibrium, if and only if the network formed in equilibrium has at least i dealers. Thus in equilibrium $\hat{l}_{i-1,i} = 1$ for $i \in \{2, 3, \ldots, \bar{n}\}$, if and only if $C \leq C^*_{i-1,i}(\kappa_1)$, where

$$C^*_{i-1,i}(\kappa_1) \equiv \max\{C^*(i), \ldots, C^*(\bar{n})\}.$$

I denote $C^*_{i-1,i,n_p}(\kappa_1)$ to be the threshold in the game without a public signal, and $C^*_{i-1,i,p}(\kappa_1)$ to be the threshold in the game with $p_{1,2}$ as the public signal.

Proposition 5. *In equilibrium of the network formation game, when κ_1 is relatively large,*

(1) for $i \in \{n_p+1, \ldots, \bar{n}\}$, $C^*_{i-1,i,n_p}(\kappa_1) > C^*_{i-1,i,p}(\kappa_1)$;

(2) for $i \in \{2, \ldots, n_p\}$, $C^*_{i-1,i,n_p}(\kappa_1) \geq C^*_{i-1,i,p}(\kappa_1)$.

Proposition 5 is directly implied by Proposition 4. When κ_1 is relatively large, for any two consecutive dealers that are affected by the public signal, due to the larger information asymmetry between them, they have lower trading profit. That lowers the threshold of cost for forming the link between them.

It also implies that TRACE could reduce the trading frequency of the upstream dealers for whom trading profits are not affected by the public signal. That is because they benefit less from the downstream dealers' trading profits if they form the link to initiate the trade. Figure 1.4 plots $C^*_{1,2,n_p}(\kappa_1)$ in the game without a public signal and $C^*_{1,2,p}(\kappa_1)$ in the game with $p_{1,2}$ as the endogenous public signal. In markets with relatively large κ_1, the public signal reduces the cost threshold of the first link's formation, and thus reduces the ex-ante probability of any inter-dealer trade.

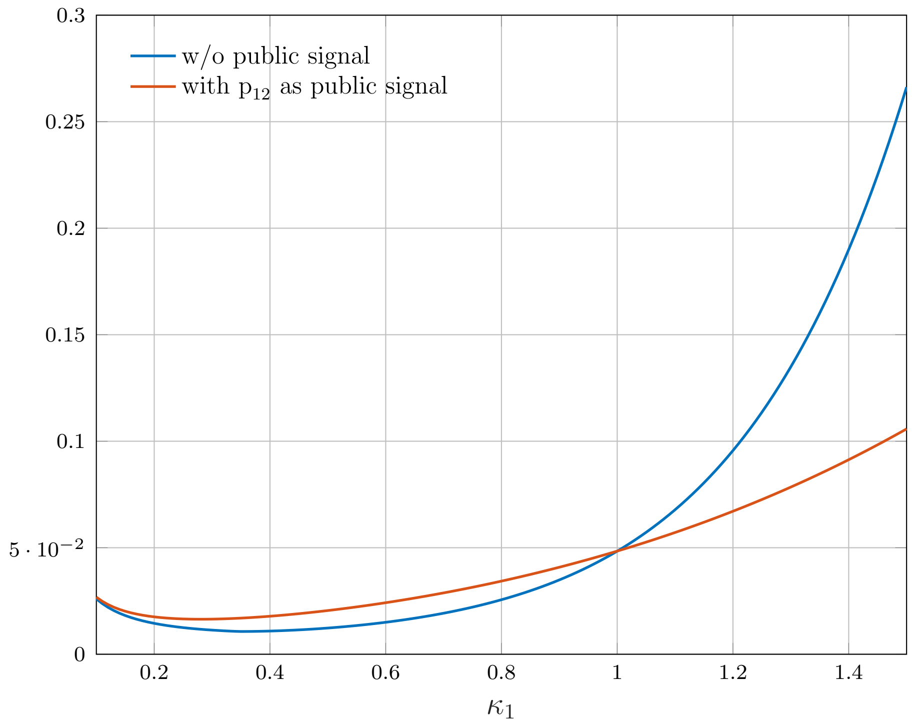

Figure 1.4: Cost Threshold for Network Formation

$C^*_{1,2,n_p}(\kappa_1)$ and $C^*_{1,2,p}(\kappa_1)$

Parameters: $\beta = -1$, $\sigma_\theta = 1, \sigma_\eta = 0.1$, $n = 5$, $n_p = 3$.

1.6 Empirical Analysis

In this section, I study the effect of TRACE on corporate bond markets, by utilizing the Academic Corporate Bond TRACE data and the Mergent FISD database. The argument is that if a bond is very actively traded, then although investors are more likely to know more about them, they are also more likely to be asymmetrically informed; vice versa, if a bond is thinly traded, then investors are more likely to be symmetrically uninformed. So the model predicts that TRACE would have negative effects on inter-dealer trading activity for thinly traded bonds.

The regression results indicate that TRACE has a significant negative effect on the inter-dealer trading frequency for thinly traded bonds. It is also a warning for the widespread implementation of TRACE, as many of the assets that are newly subject to transparency are also thinly traded.

The main difference between my empirical work and [ACP13] is that, the trading activity in their paper is the aggregate trading activity (including the trade between dealer and customer and inter-dealer trade), while mine is the inter-dealer trading activity (and further separate inter-dealer trade into principle inter-dealer trade and agency inter-dealer trade).

1.6.1 Data Description

The data source for corporate bond trading is the Academic Corporate Bond TRACE data, purchased from FINRA. During the time period, July 1, 2002 until December 31, 2005, there are 29,064,905 unique trade reports on 37,026 different CUSIPs in the Academic TRACE dataset.

The Mergent FISD database is my source for bond characteristics such as issue size, credit ratings, maturity, etc., which I add to the Academic TRACE dataset. The Mergent FISD database I use include all the bonds with an offering date between January of 1950 and January of 2010.

1.6.2 Steps from the Academic TRACE File to the Cleaned Academic TRACE Sample

There are a number of reporting errors in this self-reported data. The Appendix 4.1.3 describes the steps I take to convert the Academic TRACE File to the Cleaned Academic TRACE Sample. The references for the cleaning procedure are [ACP13] and [Dic14]. Table 1.1 reports the number of bonds and trade reports after each step.

1.6.3 Steps from FINRA's Phase Listings to the Cleaned Phase Sample

Dissemination took place in Phases over two-and-a-half years. FINRA's main criteria for a bond's dissemination Phase are the bond issue size and credit rating. Actively traded, investment grade bonds became transparent before thinly traded, high-yield bonds. Phase 1 of TRACE was implemented on July 1, 2002. Phase 2 of TRACE was implemented on March 3, 2003. Phase 3A of TRACE was implemented on October 1, 2004. Phase 3B of TRACE was implemented on February 7, 2005.

I begin with a list of all Phase 1, 2, 3A, and 3B bonds. There are 16,854 bonds in this list, of which 15,769 exist in the Cleaned Academic TRACE sample. They have 3,526,543 trades during our sample period.

In addition to the four Phases that correspond to the FINRA dissemination dates, FINRA also maintained two other lists of bonds, which we call the FINRA50 and the FINRA120[8]. The FINRA50 represent 50 Non-Investment Grade (High-Yield) securities disseminated under the Fixed Income Pricing System (FIPS)[9]. This list of 50 bonds changes over time with bonds both entering and exiting. I eliminate any bonds that also exist in the FINRA50. There are 3,517,618 unique trade reports (phase 1: 1,031,396, phase 2: 668,300, phase 3A: 1,613,748, phase 3B: 204,174) and 15,781 different CUSIPs (phase 1: 370, phase 2: 2,348, phase 3A: 10,492, phase 3B: 2,551) left.

[8]Dissemination began on April 14, 2003 for a group of 120 Investment-Grade securities rated BBB. These BBB bonds are denoted as the FINRA120.

[9]FIPS started in April 1994. It reported transactions information on approximately 50 high-yield bonds.

Table 1.1: Steps from Academic TRACE File to Cleaned Academic TRACE Sample

	Remaining CUSIPs	Remaining trade reports
Source: Academic TRACE	37,026	29,064,905
Eliminate trade btw dealer and customer	32,795	11,900,080
Eliminate bonds based on characteristics		
Bonds unmatched to FISD by CUSIP	31,682	11,800,641
Convertible bonds and exchangeable bonds	29,872	11,005,869
SEC Rule 144a bonds	26,965	10,704,976
Bonds with 0 or very small issue size	26,729	10,687,352
Eliminate trade because of self-reported errors		
Same day corrections and cancellations	26,647	10,423,151
Reversals: ten-way match	26,581	10,121,712
Reversals: nine-way match	26,578	10,113,094
Reversals: nine-way match (price rounding to 0.01)	26,578	10,112,431
Eliminate double report	25,239	5,155,102
Address trade splitting	25,239	4,889,149
Eliminate trade reports with price or volume issues		
Prices that are vastly out of line	25,239	4,885,676
Prices/volumes below 0.01% or above 99.99%	25,181	4,883,688
Eliminate trade that is under special circumstances, etc.	25,101	4,865,045

Table 1.2: Steps from FINRA's Phase Listings to the Cleaned Phase Sample

	CUSIPs	Trade reports
Sample 1: Cleaned Academic TRACE Sample	25,101	4,865,045
Sample 2: FINRA list of Phase 1-3B bonds	16,854	-
Sample 3: Bonds in both Sample 1 and 2	15,769	3,526,557
Bonds in Sample 3 but not in FINRA50	15,761	3,517,632
(Cleaned Phase Sample)		
Phase 1	370	1,031,409
Phase 2	2,348	668,300
Phase 3A	10,492	1,613,741
Phase 3B	2,551	204,182

1.6.4 Key Statistics

Different from other bond characteristics, a bond usually has multiple credit ratings that are specific to rating date. I assign credit ratings that are rated since July 2002. Data on credit ratings are from FISD. FISD includes ratings from S&P, Moody's, Fitch and Duff and Phelps. I first transform Moody's rating to be consistent with others. For the phase 2 (3A, 3B) exercise, for each bond in the credit rating data, I keep the observation with the rating date that is after and closest to July 2002.[10]

From Table 1.3, we can see Phase 3B bonds have significant lower credit ratings than bonds in other Phases; Phase 3A bonds have much smaller issue size.

For the regression in the next section, I construct three subsamples. The Phase 2

[10]As an alternative, I could use the bond's credit rating that was established after its phase began. I do not use that, because many bonds, especially Phase 3B bonds, were not rated anymore after the start of Phase 3B. For example, for the 2551 Phase 3B bonds in my sample, there are only 1126 bonds that are rated after February 7, 2005, when TRACE was implemented for Phase 3B.

Table 1.3: Bond Issue Size and Credit Rating

	Phase 2	Phase 3A	Phase 3B
Number of bonds	2,348	10,492	2,551
Number of bonds Issue size ($M)			
mean	274	88	185
p5	100	1	1
p10	100	1	5
p25	150	3	100
median	237	12	150
p75	325	92	235
p90	500	300	350
p95	600	450	450
Credit rating			
mean	A+	A-	B+
p5	AAA	AAA	BBB
p10	AA	AA	BBB-
p25	AA-	A+	BB
median	A+	A-	BB-
p75	A	BBB+	B-
p90	A-	BBB-	CCC
p95	A-	BB+	CC
# of bonds with rating information	2,200	10,334	2,142

subsample keeps the transactions from December 3, 2002 to June 3, 2003; the Phase 3A subsample keeps the transactions from July 1, 2004 to the end of 2004; the Phase 3B subsample keeps the transactions from November 7, 2004 to May 7, 2005. For each subsample, I fill in the bonds that are not observable within that subsample but are observable in other subsamples, and count their number of trades as 0; then I drop the bond if its offering day is after the policy implementation date or its maturity day is before the policy implementation date or its credit rating information is missing.

For each subsample, I calculate each bond's number of inter-dealer trade per day within 3 months before the policy implementation and 3 months after the policy implementation.[11] Table 1.4 shows the average number of trades per day for Phase 2 bonds in the Phase 2 subsample, Phase 3A bonds in the Phase 3A subsample, and Phase 3B bonds in the Phase 3B subsample. We can see Phase 3B bonds are much less frequently traded than Phase 2 and Phase 3A bonds.

Table 1.4: Number of Overall Inter-dealer Trades per Day by Phase

	num of trades per day within 3 months before	num of trades per day within 3 months after
Phase 2 bonds	0.2479	0.2857
Phase 3A bonds	0.1289	0.1286
Phase 3B bonds	0.0486	0.0509

In some trades, dealers may act as agent. In agency transactions, a dealer does intermediation by transferring the bond while not assuming any price risk, thus the agency transaction does not reflect the transfer of bond ownership or price risk. The principal inter-dealer trade is defined to be the trade between two dealers whose trading capacities are both principal. The agency inter-dealer trade is defined to be the trade between two dealers and at least one side's trading capacity is principal. I separate the over-all inter-dealer trades into agency inter-dealer trades and principal inter-dealer trades, to study the separate effects of TRACE on them.

[11]If the bond's offering day is within 3 months before the policy implementation, its number of trades per day is the total number of trades before the policy implementation divided by the number of days between its offering day and the policy implementation. If the bond's maturity day is within 3 months after the policy implementation, its number of trades per day is the total number of trades after the policy implementation divided by the number of days between the policy implementation and its maturity day.

Table 1.5 reports each bond's number of trades per day for principal inter-dealer trade by phase. We can see for all the bonds, most of their inter-dealer trades are principal inter-dealer trades. In terms of principal inter-dealer trade, Phase 3B bonds are still much less frequently traded than Phase 2 and Phase 3A bonds. Table 1.6 reports each bond's number of trades per day for agency inter-dealer trade by phase. In terms of agency inter-dealer trade, Phase 3B bonds' trading frequency is similar to Phase 3A bonds, while Phase 3A and Phase 3B bonds are much less frequently traded than Phase 3 bonds.

Table 1.5: Number of Principal Inter-dealer Trades per Day by Phase

	num of trades per day within 3 months before	num of trades per day within 3 months after
Phase 2 bonds	0.2139	0.2473
Phase 3A bonds	0.1158	0.1171
Phase 3B bonds	0.0397	0.0398

Table 1.6: Number of Agency Inter-dealer Trades per Day by Phase

	num of trades per day within 3 months before	num of trades per day within 3 months after
Phase 2 bonds	0.0394	0.0445
Phase 3A bonds	0.0167	0.0147
Phase 3B bonds	0.0124	0.0152

1.6.5 Difference-in-Differences Regression

The before-and-after comparisons in Table 1.4 do not establish that dissemination affected trading activity because there could be contemporaneous market-wide trends. I adjust for potential market trends by comparing the changes in the sample of newly disseminated bonds (the treated sample) to those that do not change dissemination status (the control sample) by estimating difference-in-differences models of the form:

$$y_{it} = \lambda_1 \text{Disseminate}_i + \lambda_2 \text{Post}_t + \lambda_3 \text{Disseminate}_i \times \text{Post}_t + \lambda_4 X_{it} + \varepsilon_{it}$$

where i denotes the bond, t denotes the time period (3 months before/after the dissemination), y_{it} is bond i's outcome (number of trades per day), Disseminate_i is an indicator for whether the bond changes dissemination status (i.e., is in the treated group), Post_t is an indicator for the trade outcomes after the dissemination, and X_{it} is a vector of bond i's characteristics (issue size and credit rating).

Any pre-existing difference between bonds that change dissemination status and those that do not are captured by λ_1. Any effects of dissemination that accrue to all bonds — that is, effects that are not limited to only bonds that change their dissemination status in the Phase — are absorbed by time effects λ_2. The coefficient of interest is λ_3, which estimates the direct effect of transparency on the outcome variable. The coefficient λ_3 reflects the change in trading outcomes for bonds that change dissemination status compared to the change in trading outcomes for bonds that do not change dissemination status. Estimates of λ_3 therefore, net out aggregate changes in bond trading outcomes.

The control bonds for Phase 2 are the disseminated bonds in Phase 1, and the non-disseminated bonds in Phase 3A and Phase 3B. For Phase 3A and Phase 3B, the control bonds are the disseminated bonds in Phase 1 and Phase 2. Phase 3A bonds are not a control for Phase 3B and vice versa because Phase 3A and Phase 3B occur just over four months apart, on October 1, 2004 and February 7, 2005, respectively. For λ_3 to provide unbiased estimates of the causal effect of transparency, I assume that the change over time in control bonds' behavior reveals what would have occurred to treated bonds if there had been no change in their dissemination status.[12]

[12]The other necessary assumptions are, first, that transparency and its consequences are not well anticipated by market participants and thus the impacts on the trading activity would not appear before

Table 1.7 reports the estimates of equation for overall inter-dealer trades of Phase 2 bonds, Phase 3A bonds and Phase 3B bonds. For Phase 2 bonds and Phase 3A bonds, TRACE post-trade transparency does not have significant effect, for Phase 3B bonds, it significantly reduces the number of trades. TRACE reduces the average number of overall inter-dealer trades for Phase 3B bonds by 0.09. This represents a 190% drop from the average level before dissemination.

Table 1.7: DID Regression Results for Overall Inter-dealer Trade

	Phase 2 num of trades per day	Phase 3A num of trades per day	Phase 3B num of trades per day
Disseminate	-0.303***	-0.205***	-0.994***
	(-10.79)	(-5.63)	(-5.09)
Post	0.0250***	-0.0151*	0.0947***
	(4.72)	(-2.00)	(3.80)
Disseminate×Post	0.0128	0.0148	-0.0924***
	(1.57)	(1.86)	(-3.69)
N	19998	24946	8418

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 1.8 reports the estimates of equation for principal inter-dealer trades. For Phase 2 bonds and Phase 3A bonds, TRACE post-trade transparency does not have a negative effect, whereas for Phase 3B bonds it significantly reduces the number of principal inter-dealer trades.

Table 1.9 reports the estimates of equation for agency inter-dealer trades. For Phase 2 bonds, TRACE post-trade transparency does not have a significant effect, whereas for Phase 3A and Phase 3B bonds it significantly reduces the number of agency inter-dealer

the actual change in dissemination status; and secondly, that there are no other changes simultaneous with the phase start date that affects the trading activity for those bonds changing dissemination status, as argued in [ACP13].

Table 1.8: DID Regression Results for Principal Inter-dealer Trade

	Phase 2 num of trades per day	Phase 3A num of trades per day	Phase 3B num of trades per day
Disseminate	-0.260***	-0.137***	-0.872***
	(-10.75)	(-4.29)	(-5.18)
Post	0.0213***	-0.0259***	0.0810***
	(4.21)	(-4.13)	(3.54)
Disseminate×Post	0.0121	0.0273***	-0.0809***
	(1.56)	(4.06)	(-3.53)
N	19882	24896	8314

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

trades. TRACE reduces the average number of agency inter-dealer trades for Phase 3A bonds by 0.0138. This represents a 83% drop from the average level before dissemination. TRACE reduces the average number of agency inter-dealer trades for Phase 3B bonds by 0.012. This represents a 97% drop from the average level before dissemination.

In summary, Phase 2 bonds are the most actively traded bonds in the overall inter-dealer trade, the principal inter-dealer trade, and the agency inter-dealer trade. DID estimates show that TRACE does not have a significant effect on the trading frequency of Phase 2 bonds. Phase 3A bonds are similarly thinly traded as Phase 3B bonds in the agency inter-dealer trade. DID estimates show that TRACE significantly lowers the trading frequency of Phase 2 bonds in agency inter-dealer trade and the magnitude of the negative effect is quantitatively similar to that on Phase 3B bonds. Phase 3B bonds are the most thinly traded bonds in the overall inter-dealer trade, the principal inter-dealer trade, and the agency inter-dealer trade. DID estimates show that TRACE significantly lowers the trading frequency of Phase 3B bonds and the overall negative effect mainly comes from the negative effect on the principle inter-dealer trade.

Table 1.9: DID Regression Results for Agency Inter-dealer Trade

	Phase 2	Phase 3A	Phase 3B
	num of trades per day	num of trades per day	num of trades per day
Disseminate	-0.0502***	-0.0763***	-0.121**
	(-7.23)	(-8.79)	(-3.11)
Post	0.00458***	0.0118***	0.0148***
	(4.76)	(4.28)	(3.43)
Disseminate×Post	0.000510	-0.0138***	-0.0120**
	(0.37)	(-4.93)	(-2.68)
N	16794	20424	7234

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

1.7 Extension

In Section 1.7.1, I study the relationship between the sequential trading game and the simultaneous trading game. The equivalence between them indicates that the modeling choice of the sequential trading game does not play any role in deciding the outcome in equilibrium, even though the sequential trading game is more natural than the simultaneous trading game.

In Section 1.7.2, I explicitly model the price-discovery process. The price discovery game shows that the equilibrium prices and quantities of my baseline model can be found via an iterative, decentralized process. It shows that my equilibrium outcome is not specific to the double auction modeling technique, which captures the repeated exchange of limit and market orders (i.e., the offer and acceptance of quotes) within a short time interval across fixed counterparties.

In Section 1.7.3, I relax the assumption for clients' trading intensity to be fixed. It shows when clients trade for the asset payoff, the information diffusion process is the same as the baseline model, but clients' trading intensity would be lower when the information asymmetry becomes larger. Thus the results in the baseline model still hold and the effect on dealers' trading surplus are amplified via changing clients' trading intensity.

1.7.1 Payoff Equivalent to the Simultaneous Trading Game

[BK18] and [BKW19] use a simultaneous trading model to study the information diffusion in the inter-dealer network. In this paper, I use the sequential trading model mainly for the sake of exposition, and it is more suitable for discussing post-trade transparency policy. Under the assumption that one dealer is informed of private information about the asset payoff and dealers are risk-neutral, the following proposition shows that the sequential trading game is payoff equivalent to the simultaneous trading game.

Proposition 6. *Considering the simultaneous trading game, where dealers' linear demand functions are*

$$\Big\{Q^1_{1,2}(\eta_1, s_1, p_{1,2}), \big(Q^2_{1,2}(\eta_2, p_{1,2}, p_{2,3}), Q^2_{2,3}(\eta_2, p_{1,2}, p_{2,3})\big), ...,$$

$$\big(Q^{n-1}_{n-2,n-1}(\eta_{n-1}, p_{n-2,n-1}, p_{n-1,n}), Q^{n-1}_{n-1,n}(\eta_{n-1}, p_{n-2,n-1}, p_{n-1,n})\big), Q^n_{n-1,n}(\eta_n, p_{n-1,n})\Big\},$$

the simultaneous trading game shown in the main text is payoff equivalent to the simultaneous trading game.

For any dealer $i \in \{2, ..., n-1\}$, the trading price on link $(i, i+1)$ is not informative about the asset payoff, as the subsequent dealers do not have private information about the asset payoff. Thus, as dealers are risk neutral, the optimization problem of trading on the link $(i-1, i)$ is separate from that on the link $(i, i+1)$, and is not affected by the price on the link $(i, i+1)$. Thus the solution to the sequential trading game I characterize above is also the solution to the simultaneous trading game.

1.7.2 The Price Discovery Game

In real-world OTC markets, dealers engage in bilateral negotiations with their counterparties by quoting prices which are valid for a certain quantity. To capture this feature, I introduce the price discovery game, as a variant of the OTC game where dealers find the equilibrium prices and quantities through a sequence of bilateral exchange of quotes. It is in the spirit of the price discovery game in [BK18].

Formally, I define the price-discovery game as follows. In round 0, dealer i chooses a bidding strategy $\pi^i_{i,i+1,0} = \{p^i_{i,i+1,0}, q^i_{i,i+1,0}\}$, and dealer $i+1$ chooses a bidding strategy $\pi^{i+1}_{i,i+1,0} = \{p^{i+1}_{i,i+1,0}, q^{i+1}_{i,i+1,0}\}$. They also choose $B^i_{i,i+1}(s_i, \pi^{i+1}_{i,i+1,\tau}) = \pi^i_{i,i+1,\tau+1}$, which describes the counter-offers that dealer i makes in round $\tau+1$, conditional on the bids it received in round τ. $B^{i+1}_{i,i+1}(s_i, \pi^i_{i,i+1,\tau}) = \pi^{i+1}_{i,i+1,\tau+1}$ describes the counter-offers that dealer $i+1$ makes in round $\tau+1$, conditional on the bids it received in round τ. If there exists a price and quantity vector $\{\bar{p}^i_{i,i+1}, \bar{q}^{i+1}_{i,i+1}\}$ with

$$\bar{p}^i_{i,i+1} = \bar{p}^{i+1}_{i,i+1},$$

$$\bar{q}^i_{i,i+1} + \bar{q}^{i+1}_{i,i+1} + \beta\bar{p}^i_{i,i+1} = 0,$$

and

$$\lim_{\tau\to\infty} \pi^i_{i,i+1,\tau} = (\bar{p}^i_{i,i+1}, \bar{q}^i_{i,i+1}),$$

for some random starting vector $\{\pi^i_{i,i+1,0}, \pi^{i+1}_{i,i+1,0}\}$, then trade takes place.

The payoff for dealer i is $\mathbb{E}[\bar{q}^i_{i,i+1}(\theta + \eta_i - \bar{p}^i_{i,i+1})|s_i]$, provided $\{\bar{p}^i_{i,i+1}, \bar{q}^{i+1}_{i,i+1}\}$ exists, and minus infinity otherwise. Thus, taking dealer $i+1$'s bidding strategy as given, dealer i solves

$$\max_{B^i_{i,i+1}(s_i,\pi^{i+1}_{i,i+1,\tau})} \mathbb{E}[\bar{q}^i_{i,i+1}(\theta + \eta_i - \bar{p}^i_{i,i+1})|s_i, \eta_i];$$

and taking dealer i's bidding strategy as given, dealer $i+1$ solves

$$\max_{B^{i+1}_{i,i+1}(\pi^i_{i,i+1,\tau})} \mathbb{E}[\bar{q}^{i+1}_{i,i+1}(\theta + \eta_i - \bar{p}^i_{i,i+1})|\eta_{i+1}].$$

The following proposition proves that dealers can find the equilibrium prices and quantities in the OTC game by playing the price-discovery game.

Proposition 7. *There exists an equilibrium in the price-discovery game, where prices and quantities are the same as the equilibrium prices and quantities in the equilibrium of the OTC game.*

1.7.3 Clients Trade for the Asset Payoff

I extend the baseline model of clients such that clients trade not only because of liquidity needs, but also for the asset payoff.

In the trade between dealer i and $i+1$, a continuum of clients participate. Clients do not have private information about the asset payoff or the dealers' private values of the asset. Clients are risk-neutral, Client j's value of the asset is $\theta + \eta_j$ with $\mathbb{E}(\eta_j) = 0$ and η_j is independently distributed across the clients on the same link. Clients incur a quadratic flow cost of trading the asset.

The demand function of client j is $q^j_{i,i+1}(p_{i,i+1}, \eta_j, S)$. The payoff of client j is

$$\mathbb{E}\big((\theta + \eta_j)q^j_{i,i+1} - p_{i,i+1}q^j_{i,i+1} - \frac{\mu}{2}(q^j_{i,i+1})^2 | p_{i,i+1}, S\big).$$

Thus we have that the demand of client j is

$$q^j_{i,i+1}(p_{i,i+1}, \eta_j, S) = \frac{1}{\mu}(\eta_j + \mathbb{E}(\theta|p_{i,i+1}, S) - p_{i,i+1}).$$

Thus the aggregate demand of clients on each link $(i, i+1)$ is $\frac{1}{\mu}(\mathbb{E}(\theta|p_{i,i+1}, S) - p_{i,i+1}) = \beta_{i,i+1}p_{i,i+1} + \delta_{i,i+1}S$.

Proposition 8. *In the Linear Bayesian Nash equilibrium of this model,*

1. the information diffusion process is the same as the Bayesian Nash equilibrium of the baseline model;

2. in the trade between dealer i and $i+1$, clients' trading intensity $\beta_{i,i+1}$ and the trading surplus from the asset allocation between dealer i and $i+1$ become smaller if and only if the information asymmetry between dealer i and $i+1$, $Var(\theta|S) - Var(\theta|\hat{s}_i, S)$ becomes larger.

Thus the qualitative results in the baseline model still hold and are amplified due to the effect of information asymmetry on clients' trading intensity.

1.8 Conclusion

The harm of TRACE has been argued by many market participants. A recent study by [ACP13] finds that after transparency, there is a significant decline in the number of trades for thinly traded bonds. Many of the securities markets that are newly subject to transparency are thinly traded. Their empirical results support the view that not every segment of a security market should be subject to the same degree of mandated transparency.

This paper provides a theory framework to fill the gap in the literature in that regard. I build up an information model that features bilateral trade in a double auction, endogenous public signal, and inter-dealer network formation to study the effect of TRACE on the inter-dealer markets. For the sake of tractability, I assume there is one dealer in the market that has private information about the asset payoff. In the trading stage, I study the private information diffusion process, and endogenize the public information contained in the disseminated trading price; I show that in markets with a relatively low degree of information asymmetry, post-trade transparency would increase the degree of information asymmetry, and thus make the adverse selection more severe and reduce the surplus from asset reallocation between dealers. The effect of TRACE on dealers' trading profit affects dealers' network decisions and for markets with a low degree of information asymmetry TRACE hurts the network formation and lowers inter-dealer trading frequency. This model implies that the effects of TRACE on inter-dealer trading frequency are not uniform across different markets with different information asymmetry, supported by the empirical evidence in Section 1.6. In recent years, TRACE-like transparency policy has been expanding in many other securities markets, e.g., the Agency-Backed Securities market and Asset-Backed Securities required by FINRA, the swaps market required by Title VII of Dodd-Frank Wall Street Reform, etc. This paper sheds light on the discussion whether besides the corporate bond market, other securities markets should be subject to TRACE-like post-trade transparency.

In future research, this framework can be applied to the study of other types of information sharing policies in the OTC markets (for example, pre-trade transparency for bonds brought by MiFID II/R's regulatory regime in Europe, more information sharing of agents' private trading needs in CDS markets[13], etc.) and cast light on their effects

[13] Congress passed Section 13 (f) of the Securities Exchange Act in 1975 in order to increase the public availability of information regarding the security holdings of institutional investors. [BST19] interpret the information of institutional investors' security holdings provided to the market in the 13-F form as information related to the trading needs of investors on CDS, as CDS indexes are a way for institutions

on agents' trading behavior and trading surplus, the information diffusion process, and network structure.

to hedge against risk in their portfolios.

CHAPTER 2

Large Traders, Information Manipulation, and Crises

2.1 Introduction

In many episodes during the currency crises, large traders were implementing "double play" strategies, that is, they short the asset markets, e.g., equity or bond market, and the currency market at the same time. For example, in South Africa in 1998, large international financial firms were taking short positions in the bond market and in the currency market; in Hong Kong in 1998 during the Asian crisis, large macro hedge funds were shorting the Hong Kong equity markets and Hong Kong dollars. [1]

We propose a stylized model to study the large trader's information manipulation and its implication for asset prices during currency crises. To isolate the forces and effects of the information manipulation, the large trader is assumed to be without private information. The forces at play are that, the dividend of the asset depends on the underlying fundamental, the equilibrium asset price will convey information about the fundamental in the coordination game; as the lower asset price indicates the worse fundamental, small traders become strictly more aggressive after observing the lower asset price, thus the lower asset price strictly increases the mass of small traders who attack the currency regime. That incentivizes the large trader to manipulate the asset price in favor of its currency attack. We show that in all the equilibrium the large trader will manipulate the asset price to be lower than the price in the case that its demand schedule in the asset

[1] As noted in Financial Stability Forum (2000), "Among those taking short positions in the equity market were four large HFs (Hedge Funds), whose futures and options positions were equivalent to around 40 percent of all outstanding equity futures contracts as of early August prior to the HKMA(Hong Kong Monetary Authority) intervention. Position data suggest a correlation, albeit far from perfect, in the timing of the establishment of the short position. Two HFs substantially increased their position during the period of the HKMA intervention. At the end of August, four hedge funds accounted for 50,500 contracts or 49% of the total open interest/net delta position; one fund accounted for one third. The group's meeting suggested that some large HLIs (Highly Leveraged Institutions) had large short positions in both the equity and currency markets."

market is observable. The intuition is that, when the large trader's position in the asset market is not observable, in a fictitious "equilibrium" with no manipulation, the large trader can manipulate the asset price to be infinitesimal lower with infinitesimal cost, while its marginal benefit in the currency-attack game is strictly bounded above zero, so no manipulation will not be the large trader's strategy in equilibrium. This result doesn't reply on whether the price manipulator is large or small in the currency-attack game.

Also, in equilibrium, the large trader's manipulating the asset price to be lower and attacking the currency regime are concurrent; the large trader's manipulation in the asset market is most significant when the public signal is in the intermediate range. The intuition is straightforward. If the large trader doesn't attack the currency regime, then its incentive to manipulate the asset price goes away; when the public signal is in the intermediate range, the marginal benefit from a lower public signal in the currency-attack game is most significant.

It's worth mentioning the mechanism above goes beyond the currency crises scenario. In bank runs, sovereign debt crisis, financial crashes and other applications captured by Morris-Shin and others using coordination game, we observe that in many scenarios, the larger trader's behavior is suspected to be information manipulation. For example, SEC once investigated the market manipulation in 2008 financial crisis.[2] As pointed out by Gary Gensler (Ex-CFTC Chair),

> "in the fall of 2008, stock prices were in a free fall...CDS figured into that decline...buyers of credit default swaps had an incentive to see a company fail, they may have engaged in market activity to help undermine an underlying company's prospects."

The second stage in the above setting is a bank-run or debt roll-over scenario. The lower equity price would lead more creditors to ask the company for debt repayment, and also make the company harder to roll over its debt, thus increase the probability of the company's failure. That incentivizes the large investors in the equity market, who hold the CDS, to manipulate the equity price to be lower. Another scenario is the manipulation of the deficit and debt statistics by the Greek government in 2000s. The

[2]See "Anatomy of the Morgan Stanley Panic" (https://www.wsj.com/articles/SB122748970896452051)

lower deficit and debt statistics makes more debtors willing to lend money or roll over the sovereign debt.

Our results imply that the manipulator is worse off when it has the manipulation power than it does not have. So the manipulator is trapped by its manipulation power. This is a surprising result in the first sight, but makes great sense considering that in the rational expectation equilibrium nobody is fooled, the manipulation is costly and the manipulation is the case in all the equilibrium.

Finally we draw policy implication regarding the market transparency. After the crisis, it's commonly believed and advocated that the complete transparency helps stabilize the currency regime.[3]

We show that if the asset market is transparent, a natural equilibrium refinement that incorporates forward induction reasoning selects the equilibrium where all the traders behave most aggressively in the currency-attack game. The public understand that the large trader could have easily coordinated an aggressive attack by taking a non-trivial position in the asset market, and hence the most aggressive attack is coordinated even when the large trader does not do so. It highlights how the transparent asset market as large traders' coordination device for aggressive currency attack, could jeopardize the currency regime. That is in contrast with the traditional wisdom that transparency helps stabilize the currency regime.

The remainder of the introduction contains the market features and literature review. The article is then structured as follows. Section 2.2 presents the baseline model with the first-stage asset market and second-stage currency-attack game. The asset market is not transparent, in the sense that the large trader's demand schedule is not observable for small traders. Section 2.3 presents the equilibrium definition and characterizes the multiple equilibrium when the first-stage large trader and the second-stage large trader are two different large traders. The multiplicity of equilibrium arises from the strategic

[3]The Hong Kong Financial Secretary, Mr Donald Tsang, said new international rules and regulations on transparency and disclosure should cover not only government accounts but all foreign exchange movements by investment banks and hedge funds. Quote from his speech in 1998, "There must be total and complete transparency from everyone. What is the point if governments are totally open and accountable but private funds are not?...We need greater transparency from government and fund managers so investors can make educated and sensible decisions based on sound economic fundamentals and data."

complementarity between the second-stage large trader and small traders, and the large trader is large in the currency market. Section 2.4 shows if the one large trader participates in the asset market and currency-attack game, under some assumption to ensure the single-crossing property, the equilibrium exists and there are multiple equilibrium. In all the equilibrium, the incentive of the large trader to manipulate the asset price leads to financial contagion in the asset markets and the large trader is worse off than in the case that its demand schedule in the asset market is observable. Section 2.5 shows if the asset market is transparent, a natural equilibrium refinement that incorporates forward induction reasoning would select the equilibrium where every trader behaves most aggressively in the currency-attack game and the currency regime is most fragile. Policy implications regarding the market transparency is discussed. Section 2.6 concludes. The proofs are shown in the appendix.

2.1.1 Market Features

Firstly, the markets where large traders operated were opaque, as emphasized by the Chief Executive of the Hong Kong Monetary Authority, Joseph CK Yam in 1999, "Only these hedge funds have the knowledge of the size of their very large positions." This is especially the case for the currency market as most foreign exchange took place in OTC market, "The OTC markets in which hedge funds normally operate are equally opaque."

Secondly, it's widely accepted that the foreign investors have information disadvantage relative to the locals. There is a branch of literatures showing the foreign investors' information disadvantage relative to the locals. That information asymmetry between domestic and foreign investors is the most convincing explanation for the home bias phenomenon.[4][5]

In our model presented later, we assume the large trader, as foreign investor, does not

[4] Home bias refers to the phenomenon that, despite the well documented gains from international diversification, investors continue to show a strong preference for investing in domestic assets.

[5] Please see [Low93], [Geh93], and [Kan97]. The alternative explanations such as barriers to capital flows created by higher costs of transacting in foreign securities, withholding taxes, and political risk, the failure of purchasing power parity (PPP) and regulation, are negated by the findings in [TW95], [CK94], [FP91], [Fra03]. [BC97] examines U.S. portfolio investment in emerging markets and finds strong evidence that is consistent with U.S. investors being at an informational disadvantage relative to locals in emerging markets, and trading on new information with a lag. [VV09] shows why global information access does not eliminate this asymmetry.

have private information. This assumption will not hold perfectly in reality, but was made to deliver the theoretical insights in the most transparent way. This assumption makes the model tractable, and isolates the large trader's information manipulation incentive in a clear way, which will be shown later.

2.1.2 Related Literature

Following [MS98], the backbone of our model is the coordination game that Stephen Morris, Hyun Song Shin and others have applied to explain currency crisis, bank runs and financial crashes. Such game is usually called global game.[6]

Our build-up of asset market is similar with [AW06]. They employ a two-stage model, incorporating a first-stage asset market and a second-stage coordination game of currency attack, with continuum of small traders participating in both of them, to figure out the role of information in currency crisis. In terms of asset market, their model setup, equilibrium definition and characterization are built upon [GS76]. [AW06] emphasizes the role of asset market in producing public signal. Their main finding is that with endogenous public signal, multiplicity is ensured if private signal is relatively precise enough. Our model incorporates the large trader, thus the model setup and equilibrium in [AW06] turns out to be a degenerate case in our model when we set the measure of the large trader to be 0. In contrast with [AW06], the dimension of multiplicity this paper focuses on arises from the strategic complementarity between the large trader and small traders, and the large trader is large in the currency market.

Our construction of asymmetric coordination game is similar with [CDM04] but with different information structure. [CDM04] aims at pinpointing the effect of a large trader, Soros, on other small traders' equilibrium strategy, in a one-stage coordination game of currency attack. The effect is contingent on the size of Soros, precision of Soros' private signal, etc. In [CDM04], public signal is not considered, and any trader, including Soros, only observes his private signal.

Our paper is also linked to the literature about price manipulation in the financial markets, see [AG92], [AG91], [CY04]. The manipulation incentive in our paper is different

[6] Also, global game and quasi-global game frame are also used for the analysis of political issue, e.g. [Edm13], [De 10]

from the previous literature.

In game theory, forward induction has already been talked about in many literatures. A typical example about forward induction is "burn money", which is referred to in [BD92], [Van89]. It highlights the key point that, in the equilibrium surviving the refinement featured by forward induction reasoning, the option of "burning money" won't be executed but the potential of "burning money" guarantees the best outcome for the player with this potential. In our model, if the asset market is transparent, the equilibrium surviving our equilibrium refinement indicates a unique equilibrium outcome that is most in favor of the large trader, where its potential to "burn money" in asset market is not executed, but drives all the traders to be most aggressive in the currency-attack game, and hence jeopardizes the stability of the currency regime.

2.2 Model Setup

In this paper, we build up a two-stage model. In the first-stage, there is an asset market, where a large trader and a group of small traders trade a risky asset. In the second-stage, there is a coordination game of currency attack, where a large trader and another group of small traders participate.

2.2.1 The First-stage Asset Market

In the asset market, there is a measure ω continuum of small traders, indexed by $i \in [0, 1]$, and there is a measure-one larger trader. Nature draws θ from an improper uniform distribution over the real line.[7] Each small trader receives private signal $x_i = \theta + \sigma_x \xi_i$, where $\sigma_x > 0$ and $\xi_i \sim N(0, 1)$ is independent of θ, i.i.d across small traders. $\alpha_x := \sigma_x^{-2}$ denotes the precision of small traders' private signal. The large trader doesn't receive private signal.

The risky asset is with dividend $f = \theta$.[8] The price of the asset is determined in an

[7] The setup for improper uniform prior of θ follows [CDM04], [AW06] and [OY08].

[8] Besides $f = \theta$, [AW06] also discusses the case where the asset's dividend is endogenously determined by the coordination game. Extending our model to that case complicates the analysis, while does not bring new insight.

auction. Small traders and the large trader submit their menu of prices and quantities of assets they are willing to purchase or sell at each price, $k_i(p)$, $z(p)$. $z(p)$ is not observable for small traders.

The supply of the asset is uncertain,

$$K^s(\varepsilon) = \sigma_\varepsilon \varepsilon,$$

where $\sigma_\varepsilon > 0$, $\varepsilon \sim N(0,1)$, independent of θ and ξ_i.

The auctioneer specifies a market-clearing price p that equates aggregate demand and supply, i.e., $\int_i k_i(p) + z(p) = K^s(\varepsilon)$.

The large trader is risk neutral.[9] The utility of small trader i is,

$$V(w_i) = -e^{-\gamma w_i},$$

where risk aversion parameter $\gamma > 0$, final wealth $w_i = w_0 - pk_i + fk_i$.

2.2.2 The Second-stage Currency-attack Game

In this stage, there is a continuum of small traders, indexed by $j \in [0,1]$, and a large trader. The distinguishing feature of the large trader is that he has access to a sufficiently large line of credit in the domestic currency to take a short position up to the limit of $\lambda < 1$. All small traders taken together have a combined trading limit $1 - \lambda$.

Small trader j receives private signal $x_j = \theta + \sigma_x \xi_j$, where $\sigma_x > 0, \xi_j \sim N(0,1)$ is independent of θ, i.i.d across small traders. The large trader doesn't receive private signal. All traders in this stage can observe the market clear price of the asset in the asset market.

Each small trader j can choose between two actions, either attack the status quo $a_j = 1$, or not $a_j = 0$. The large trader can choose to attack $a_s = 1$, or not $a_s = 0$. The cost of attack is denoted by $t \in (0,1)$. t is normalized relative to the payoff to a

[9]In dealing with heterogeneous traders in the asset market, like [LM90] and [OY08], we assume one group of traders are risk averse, and other traders (the large trader in this paper) are risk neutral. From empirical side, Blume, M.E.,J. [BCF74] shows that the average degree of portfolio diversification is roughly equivalent to having an equally weighted portfolio with two stocks; quote from [AG92], "Most of the stockholders are small...The fraction of their wealth held in this stock is large enough to make them risk averse." "their wealth (large traders' wealth) is sufficiently large enough that they hold many stocks and are well diversified. As a result, they can be treated as approximately risk neutral."

successful attack, so that the payoff to a successful attack is given by 1, and the payoff from refraining from attack is given by 0.

In the end of this stage, the status quo is abandoned if the mass of attackers is larger than θ, i.e., $A := \int_j a_j + a_s > \theta$, and the status quo is maintained otherwise[10] It follows that the payoff of agent j is

$$U(a_j, A, \theta) = a_j(1_{A>\theta} - t).$$

2.3 "Two Large Traders" Model

In this part, we assume the first-stage large trader and second-stage large trader are two different traders. By doing so, we completely suppress the larger trader's incentive to manipulate the asset price. This case is a useful benchmark.

2.3.1 Equilibrium in the First-stage Asset Market

Definition 3. *First-stage equilibrium is a set of functions*

$$\left\{p(\theta, \varepsilon, z), k(x, p), K(\theta, p), z(p)\right\},$$

such that

i. Utility maximization: k and z solve the maximization problem of small traders and the large trader conditional upon their information

$$k(x, p) \in \arg\max_{k \in R} E(u(w_0 + (\theta - p)k)|x, p),$$

and

$$K(\theta, p) = E(k(x, p)|\theta, p)\omega,$$

$$z(p) \in \arg\max_{z(p)} E((\theta - p)z(p)).$$

ii. Market clearing: For all pairs (θ, ε), the price $p(\theta, \varepsilon)$ equates supply and demand

$$K(\theta, p) + z(p) = K^s(\varepsilon) = \sigma_\varepsilon \varepsilon.$$

[10][AHP06] sets the model where policy maker can change the cost of attack for agents, and discusses the equilibrium in this signalling game. Different from their focus on the strategic interaction between policy maker and traders, our paper focuses on the strategic interaction between the large trader, and small traders. Thus following [MS98], [CDM04], [AW06], etc, the attack cost for agents is exogenous in our model and the authority is mechanical.

Small trader makes his optimal investment decision k to maximize his expected payoff in asset market, contingent on his information, including private signal x, price he observes p. Small traders' total demand K is the sum of all the small trader's individual demand. First-stage large trader decides the optimal demand schedule $z(p)$.

Traders in the first-stage asset market hold self-fulfilled belief about the price they observe and $z(p)$. In this paper, we focus on linear price function, that is, price is linear in θ and ε.

Proposition 9. *In linear equilibrium in the first-stage asset market,*

$$z(p) = 0, \forall p,$$

$$p = \theta - \frac{\gamma\sigma_\varepsilon}{\omega\alpha_x}\varepsilon.$$

The equilibrium characterization procedure is shown in the Appendix 4.2.1.

2.3.2 Equilibrium in the Second-stage Currency-attack Game

Definition 4. *Second-stage equilibrium is a set of functions* $\{a_j(x,p), a_s(p), A(\theta,p)\}$*, such that given* $p = \theta - \frac{\gamma\sigma_\varepsilon}{\omega\alpha_x}\varepsilon$*,*

$a_j(x,p)$ *solves the maximization problem of small trader* j*,*

$$a_j(x,p) \in \arg\max_{a_j \in \{0,1\}} E(u(a_j, A(\theta,p), \theta)|x,p),$$

$a_s(p)$ *solves the optimization problem of the large trader,*

$$a_s(p) \in \arg\max_{a_s \in \{0,1\}} E(u(a_s, A(\theta,p), \theta)|p),$$

and

$$A(\theta,p) = (1-\lambda)E(a_j(x_j,p)|p,\theta,z) + \lambda a_s(p),$$

where

$$u(a_j, A, \theta) = a_j(1_{A>\theta} - t),$$

$$u(a_s, A, \theta) = a_s(1_{A>\theta} - t).$$

We focus on monotone equilibrium, that is, for the given price realized in the first-stage asset market, small trader attacks if and only if his private signal $x_j \leq \bar{x}^*(p)$ when $p \leq p^*$, or $x_j \leq \underline{x}^*(p)$ when $p > p^*$; the large trader attacks if and only if asset price $p \leq p^*$.

Suppose $p \leq p^*$, in equilibrium second-stage large trader attacks, small traders attack if and only if $x \leq \bar{x}^*(p)$.

Define $\bar{\theta}(p)$ by

$$\lambda + (1-\lambda)Pr(x_j < \bar{x}^*(p)|\bar{\theta}(p)) = \bar{\theta}(p),$$

$$\Rightarrow \lambda + (1-\lambda)\Phi(\frac{\bar{x}^*(p) - \bar{\theta}(p)}{\sigma_x}) = \bar{\theta}(p), \tag{2.1}$$

that is, when the large trader attacks and small traders take the strategy $\bar{x}^*(p)$, status quo is abandoned if and only if $\theta \leq \bar{\theta}(p)$.

For traders in the second-stage currency-attack game, they observe asset price and take this public signal into consideration when forming their posterior of θ.

Indifferent conditions for small traders is

$$Pr(\theta \leq \bar{\theta}(p)|\bar{x}^*(p), p) = t.$$

All traders adopt Bayesian rule to update their posterior of θ, thus

$$\Phi(\sqrt{\alpha_x + \alpha_p}(\bar{\theta}(p) - \frac{\alpha_x}{\alpha}\bar{x}^*(p) - \frac{\alpha_p}{\alpha}p)) = t. \tag{2.2}$$

Combining equations (2.1)(2.2), we can pin down $\bar{x}^*(p)$ and $\bar{\theta}(p)$.

Assumption 1. *σ_x and σ_p satisfy*

$$\frac{\sigma_x}{\sigma_p^2} < \frac{\sqrt{2\pi}}{1-\lambda}.$$

Under Assumption 1, $\bar{x}^*(p)$ and $\bar{\theta}(p)$ are functions of p.

Suppose $p > p^*$, in equilibrium second-stage large trader doesn't attack, small traders attack if and only if $x \leq \underline{x}^*(p)$.

Define $\underline{\theta}(p)$ by

$$(1-\lambda)\text{Pr}(x_j < \underline{x}^*(p)|\underline{\theta}(p)) = \underline{\theta}(p),$$

$$\Rightarrow (1-\lambda)\Phi(\frac{\underline{x}^*(p)-\underline{\theta}(p)}{\sigma_x}) = \underline{\theta}(p), \tag{2.3}$$

that is, when the second-stage large trader doesn't attack and small traders take the strategy $\underline{x}^*(p)$, status quo is abandoned if and only if $\theta \leq \underline{\theta}(p)$.

Indifferent conditions for the marginal small trader is

$$\Pr(\theta \leq \underline{\theta}(p)|\underline{x}^*(p), p) = t$$

$$\Rightarrow \Phi(\sqrt{\alpha_x+\alpha_p}(\underline{\theta}(p) - \frac{\alpha_x}{\alpha}\underline{x}^*(p) - \frac{\alpha_p}{\alpha}p)) = t. \tag{2.4}$$

Combining equations (2.3)(2.4), we can pin down $\underline{x}^*(p)$ and $\underline{\theta}(p)$. Under Assumption 1, $\underline{x}^*(p)$ and $\underline{\theta}(p)$ are functions of p.

Define $\hat{\theta}(p)$ by

$$\lambda + (1-\lambda)\Pr(x_j < \underline{x}^*(p)|\hat{\theta}(p)) = \hat{\theta}(p),$$

$$\Rightarrow \lambda + (1-\lambda)\Phi(\frac{\underline{x}^*(p)-\hat{\theta}(p)}{\sigma_x}) = \hat{\theta}(p),$$

that is, when the large trader attacks, and small traders take strategy $\underline{x}^*(p)$, status quo is abandoned if and only if $\theta \leq \hat{\theta}(p)$.

Define p_1, p_2 by

$$\Pr(\theta \leq \bar{\theta}(p_1)|p_1) = \Phi(\frac{\bar{\theta}(p_1)-p_1}{\sigma_p}) = t, \tag{2.5}$$

$$\Pr(\theta \leq \hat{\theta}(p_2)|p_2) = \Phi(\frac{\hat{\theta}(p_2)-p_2}{\sigma_p}) = t. \tag{2.6}$$

Proposition 10. *Equations (2.5) (2.6) uniquely characterize p_1 and p_2. We have*

$$p_2 < p_1.$$

In Figure 2.1, the red line denotes the expected payoff of the second-stage large trader when small traders' strategy is $\bar{x}^*(p)$, that is, $\Pr(\theta \leq \bar{\theta}(p)|p) - t$; the blue line denotes the expected payoff of the second-stage large trader when small traders' strategy is $\underline{x}^*(p)$, that is, $\Pr(\theta \leq \hat{\theta}(p)|p) - t$.

Figure 2.2 depicts the large trader's expected payoff in the second-stage currency-attack game.

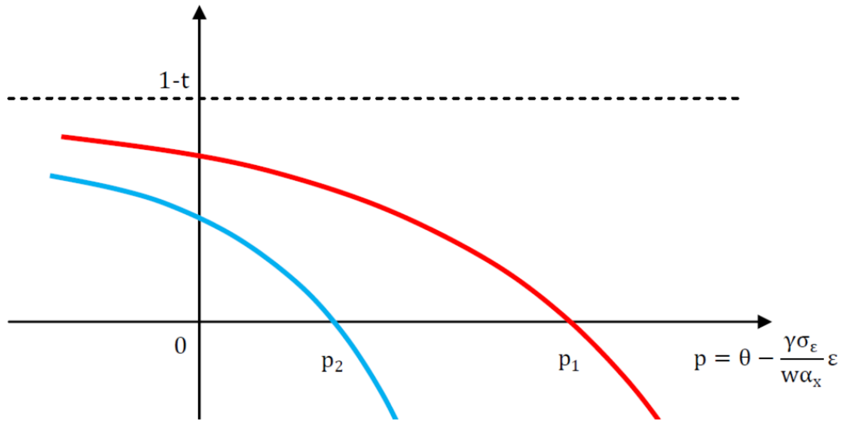

Figure 2.1: The Large Trader's Expected payoff in the Second-stage Currency-attack Game given Small Traders' Different Strategies

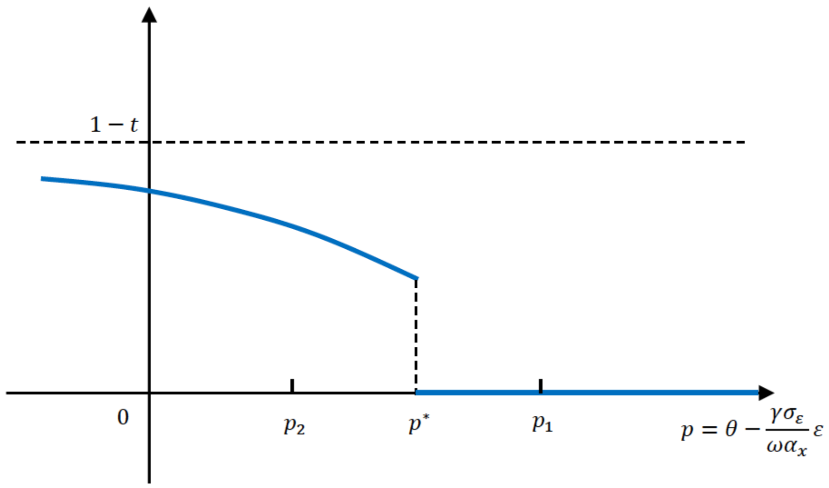

Figure 2.2: The Large Trader's Expected Payoff in the Second-stage Currency-attack Game

Proposition 11. *Any $p^* \in [p_2, p_1]$ identifies a monotone equilibrium, where*

i. the second-stage large trader attacks if and only if $p \leq p^*$;

ii. small trader attacks if and only his private signal $x_j \leq \bar{x}^*(p)$ when $p \leq p^*$, or $x_j \leq \underline{x}^*(p)$ when $p > p^*$.

2.4 "One Large Trader" Model

2.4.1 The First-stage Asset Market

The large traders in the two stages are the same trader. As shown in the Appendix 4.2.1, in equilibrium,

$$p = \theta - \frac{\gamma\sigma_\varepsilon}{\omega\alpha_x}\varepsilon + \frac{z(p)\gamma}{\omega\alpha},$$

where $\sigma_p := \frac{\gamma\sigma_\varepsilon}{\omega\alpha_x}$, $\alpha_p := \sigma_p^{-2}$, $\alpha := \alpha_x + \alpha_p$.

Small traders hold belief $z^e(p)$ of the large trader's demand schedule.

$$\tilde{p} := p - \frac{\gamma z^e(p)}{\omega\alpha}.$$

Small trader i's asset demand is,

$$k(x_i, p) = \frac{E[\theta|x_i, p, z^e(p)] - p}{\gamma Var[\theta|x_i, p, z^e(p)]} = \frac{\alpha_x x_i + \alpha_p\tilde{p} - \alpha p}{\gamma}.$$

The aggregate demand of small traders in the asset market is,

$$K(\theta, p) = E(k(x_i, p)|\theta, p)\omega = \frac{\alpha_x\theta + \alpha_p\tilde{p} - \alpha p}{\gamma}\omega.$$

Market clear condition,

$$K(\theta, p) + z(p) = \sigma_\varepsilon\varepsilon.$$

It implies

$$z(p) = M + Dp - B\tilde{p},$$

where $M := \sigma_\varepsilon\varepsilon - \frac{\omega\alpha_x}{\gamma}\theta$, $D := \frac{\omega\alpha}{\gamma}$, $B := \frac{\omega\alpha_p}{\gamma}$.

Conditional on M, the larger trader's expected payoff in the asset market,

$$U_1 = \eta(E[\theta|M] - p)z(p) = \eta(\frac{M}{B-D} - p)(M + Dp - B\tilde{p}),$$

where η normalizes the large trader's first-stage payoff, such that the payoff of a successful currency attack is 1.

2.4.2 The Second-stage Currency-attack Game

As before, we still adopt the monotone equilibrium. In this stage, small traders attack if and only if $x_i < \bar{x}^*(\tilde{p})$ when $\tilde{p} \le \tilde{p}^*$, or $x_i < \underline{x}^*(\tilde{p})$ when $\tilde{p} > \tilde{p}^*$. When, $\tilde{p} \le \tilde{p}^*$, the large trader attacks, the regime collapses if and only if $\theta < \bar{\theta}(\tilde{p})$; when $\tilde{p} > \tilde{p}^*$, the large trader doesn't attack, the regime collapses if and only if $\theta < \underline{\theta}(\tilde{p})$.

$\{\bar{x}^*(\tilde{p}), \bar{\theta}(\tilde{p})\}$ are characterized by equations,

$$\lambda + (1-\lambda)\Phi(\frac{\bar{x}^*(\tilde{p}) - \bar{\theta}(\tilde{p})}{\sigma_x}) = \bar{\theta}(\tilde{p}),$$

$$\Phi(\sqrt{\alpha}(\bar{\theta}(\tilde{p}) - \frac{\alpha_x}{\alpha}\bar{x}^*(\tilde{p}) - \frac{\alpha_p}{\alpha}\tilde{p})) = t,$$

together,

$$\alpha_p\bar{\theta}(\tilde{p}) - \sqrt{\alpha_x}\Phi^{-1}(\frac{\bar{\theta}(\tilde{p}) - \lambda}{1-\lambda}) - \alpha_p\tilde{p} = \Phi^{-1}(t)\sqrt{\alpha}. \tag{2.7}$$

Under Assumption 1, $\bar{\theta}(\tilde{p})$ is a function of $\tilde{p}$.

$$\frac{d\bar{\theta}(\tilde{p})}{d\tilde{p}} = \frac{\alpha_p}{\alpha_p - \frac{\sqrt{\alpha_x}}{1-\lambda}\frac{1}{\phi(\Phi^{-1}(\frac{\bar{\theta}(\tilde{p})-\lambda}{1-\lambda}))}}. \tag{2.8}$$

$\{\underline{x}^*(\tilde{p}), \underline{\theta}(\tilde{p})\}$ are characterized by equations,

$$(1-\lambda)\Phi(\frac{\underline{x}^*(\tilde{p}) - \underline{\theta}(\tilde{p})}{\sigma_x}) = \underline{\theta}(\tilde{p}),$$

$$\Phi(\sqrt{\alpha}(\underline{\theta}(\tilde{p}) - \frac{\alpha_x}{\alpha}\underline{x}^*(\tilde{p}) - \frac{\alpha_p}{\alpha}\tilde{p})) = t.$$

Define $\{\tilde{p}_1, \tilde{p}_2\}$ by equations,

$$\Phi(\frac{\bar{\theta}(\tilde{p}_1) - \tilde{p}_1}{\sigma_p}) = t,$$

$$\Phi(\frac{\hat{\theta}(\tilde{p}_2) - \tilde{p}_2}{\sigma_p}) = t,$$

where

$$\lambda + (1-\lambda)\Phi(\frac{\underline{x}^*(\tilde{p}) - \hat{\theta}(\tilde{p})}{\sigma_x}) = \hat{\theta}(\tilde{p}).$$

Obviously $\tilde{p}_1 = p_1, \tilde{p}_2 = p_2$. We have $\tilde{p}_2 < \tilde{p}_1$.

When small traders' strategy is $\bar{x}^*(\tilde{p})$, the large trader's expected payoff from attack in the currency-attack game is,

$$U_2 = \Phi(\frac{\bar{\theta}(\tilde{p}) - \frac{M}{B-D}}{\sigma_p}) - t.$$

2.4.3 Characterizing the Price Locus in Equilibrium

We first look at the case when $\frac{M}{B-D} \leq \tilde{p}^*$, for some $\tilde{p}^*$.

Choosing $z(p)$ is equivalent to choosing $(\tilde{p}, p)$ on $\tilde{p} = p - \frac{\gamma z^e(p)}{\omega\alpha}$. Suppose the large trader is constrained to choose $\tilde{p} \leq \tilde{p}^*$, then the large trader's constrained optimization problem is,

$$\max_{\{\tilde{p},p\}} \eta(\frac{M}{B-D} - p)(M + Dp - B\tilde{p}) + \Phi(\frac{\bar{\theta}(\tilde{p}) - \frac{M}{B-D}}{\sigma_p}) - t,$$

subject to $\tilde{p} = p - \frac{\gamma z^e(p)}{\omega\alpha}$, $\tilde{p} \leq \tilde{p}^*$.

Assuming p is a function of $\tilde{p}$,

$$U(M, \tilde{p}) := \eta(\frac{M}{B-D} - p(\tilde{p}))(M + Dp(\tilde{p}) - B\tilde{p}) + \Phi(\frac{\bar{\theta}(\tilde{p}) - \frac{M}{B-D}}{\sigma_p}) - t,$$

$$\tilde{p}^*(M) := \arg\max_{\tilde{p}} U(M, \tilde{p}).$$

In equilibrium,

$$\tilde{p}^*(M) = \frac{M}{B-D}, \qquad \forall M \geq (B-D)\tilde{p}^*. \tag{2.9}$$

A necessary condition for (2.9) to hold is,

$$\frac{\partial U(M,\tilde{p})}{\partial \tilde{p}}|_{\tilde{p}=\frac{M}{B-D}} = 0,$$

$$\Rightarrow \frac{dp}{d\tilde{p}} = \frac{(p-\tilde{p})B\eta + \phi(\frac{\bar{\theta}(\tilde{p})-\tilde{p}}{\sigma_p})\frac{d\bar{\theta}(\tilde{p})}{d\tilde{p}}\frac{1}{\sigma_p}}{(p-\tilde{p})2\eta D}, \tag{2.10}$$

where $\bar{\theta}(\tilde{p})$ is given by (2.7), $\frac{d\bar{\theta}(\tilde{p})}{d\tilde{p}}$ is given by (2.8).

From necessary condition $\frac{\partial^2 U(M,\tilde{p})}{\partial \tilde{p}^2}|_{\tilde{p}=\frac{M}{B-D}} \leq 0$, we have $p(\tilde{p}) < \tilde{p}$ for $\tilde{p} < \tilde{p}^*$; $U(M, \tilde{p})|_{\frac{M}{B-D}=\tilde{p}^*, \tilde{p}=\tilde{p}^*} = 0$, we solve p_0.

Given initial condition $(\tilde{p}^*, p_0)$, $\tilde{p}^* \geq p_0$, (2.10) pins down a locus $p(\tilde{p})$, $\tilde{p} \leq \tilde{p}^*$.

Proposition 12. *Under Assumption 2 in the Appendix 4.2.1 (i.e., σ_ε is relatively large)*[11], *for $\tilde{p}^* \in [\tilde{p}_2, \tilde{p}_1]$ such that*

$$-\sqrt{\frac{\Phi(\frac{\bar{\theta}(\tilde{p}^*)-\tilde{p}^*}{\sigma_p}) - t}{\eta D}} \geq \frac{\phi(\frac{\bar{\theta}(\tilde{p}^*)-\tilde{p}^*}{\sigma p})\frac{d\bar{\theta}(\tilde{p})}{d\tilde{p}}|_{\tilde{p}=\tilde{p}^*}}{(\frac{1}{2} - \frac{B}{4D})2\eta D\sigma_p}, \tag{2.11}$$

[11] This ensures the single-crossing property (Spence-Mirrlees Property) holds for $U(M, \tilde{p})$.

$p(\tilde{p})$ characterized below is the equilibrium locus.

$$p(\tilde{p}) = \begin{cases} p(\tilde{p}) \text{ pinned down by (2.10) given } (\tilde{p}^*, \tilde{p}^* - \sqrt{\frac{\Phi(\frac{\bar{\theta}(\tilde{p}^*)-\tilde{p}^*}{\sigma_p})-t}{\eta D}}) & \text{for } \tilde{p} \leq \tilde{p}^*; \\ \tilde{p} \quad \text{for } \tilde{p} > \tilde{p}^*. \end{cases}$$

Proposition 13. *Under Assumption 2, there are multiple equilibria.*

In Figure 2.3, the red line denotes the equilibrium locus $p(\tilde{p})$ in equilibrium $\tilde{p}^*$, the blue line denotes the 45 degree line.

It's clear that $z(\tilde{p}) < 0, \forall \tilde{p} \leq \tilde{p}^*$, which is consistent with our observation that during crisis the large trader shorts the asset market. In Figure 2.4, the red line denotes the large trader's bidding function $z(p)$ in equilibrium $\tilde{p}^*$. The incentive of the large trader to manipulate the asset price leads to financial contagion in the asset markets.

The take-away message from Figure 2.4 is, in equilibrium the large trader's shorting the asset market and attacking the currency regime are concurrent; the large trader's short position in the asset market is most significant when the public signal is in the intermediate range. Intuitively, when the public signal is very strong, manipulating the asset price in order to drive the small traders to be aggressive in currency attack is too costly for the large trader; when the public signal is very weak, the marginal benefit of depressing the asset price is small.[12]

In Figure 2.5, the red line denotes the large trader's expected payoff in the second stage game, the blue line denotes the large trader's total expected payoff. The large trader is loosing money in the first-stage asset market due to the costly manipulation.

Proposition 14. *Given any $\tilde{p}^* \in [p_2, p_1)$, there exists $\underline{\eta}(\tilde{p}^*)$, such that if $\eta \leq \underline{\eta}(\tilde{p}^*)$, $\tilde{p}^*$ is an equilibrium; there exists $\bar{\eta}(\tilde{p}^*)$, such that if $\eta > \bar{\eta}(\tilde{p}^*)$, $\tilde{p}^*$ is not an equilibrium. Thus larger η (weakly) shrinks the equilibrium set.*

[12]The manipulation behavior of the large trader is attributed to the currency peg. To see this, suppose in the second stage, the domestic currency agents trade is modeled as an asset with payoff θ. The second-stage currency market is symmetric with the first-stage asset market, but the traders in the second stage can observe the first-stage asset price. In this case, it can be proved the large trader doesn't participate in the first-stage asset market in equilibrium. The proof is available upon request.

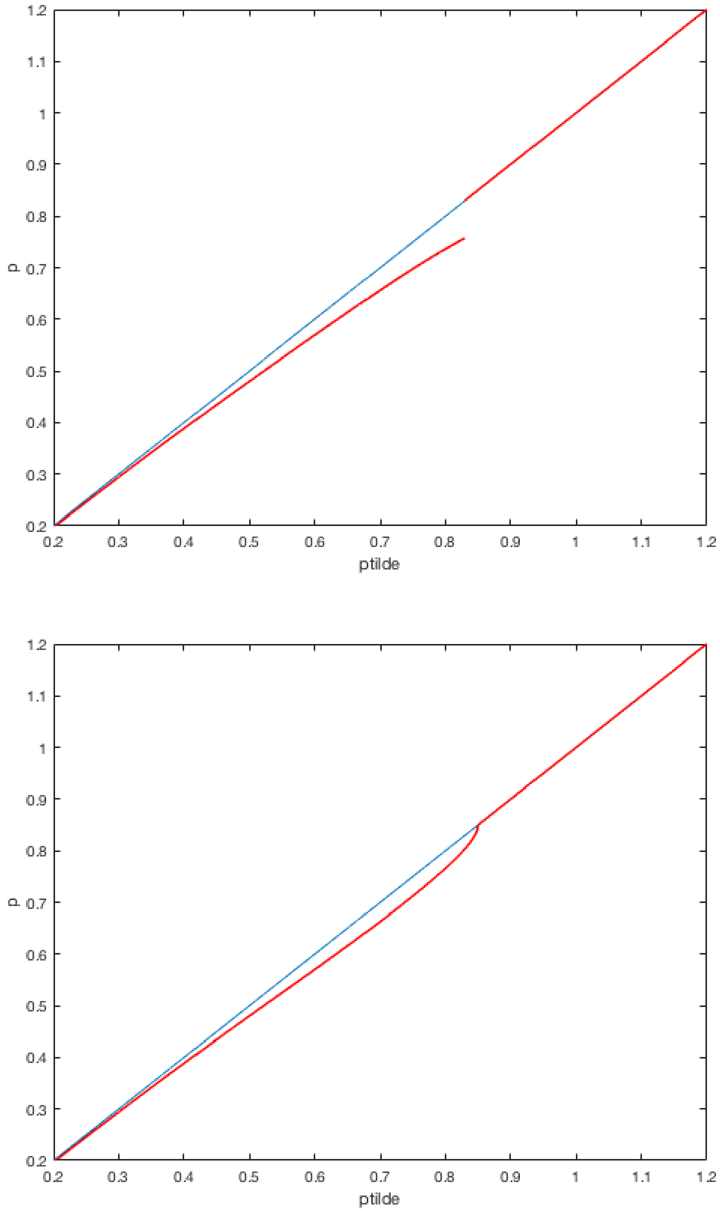

Figure 2.3: The Equilibrium Price Locus

Parameters: $\sigma_x = 0.5, \sigma_\varepsilon = 1, \omega = 1, \gamma = 2, t = 0.5, \eta = 1, \lambda = 0.7, \tilde{p}_1 = 0.85, \tilde{p}_2 = 0.7011$.

On the upper graph, $\tilde{p}^* = \frac{\tilde{p}_1 + \tilde{p}_2}{2}$; on the lower graph, $\tilde{p}^* = \tilde{p}_1$.

The proof of Proposition 14 is shown in the Appendix 4.2.1. Figure 2.6 depicts $\underline{\eta}(\tilde{p}^*)$ and $\bar{\eta}(\tilde{p}^*)$ for some parameter values.

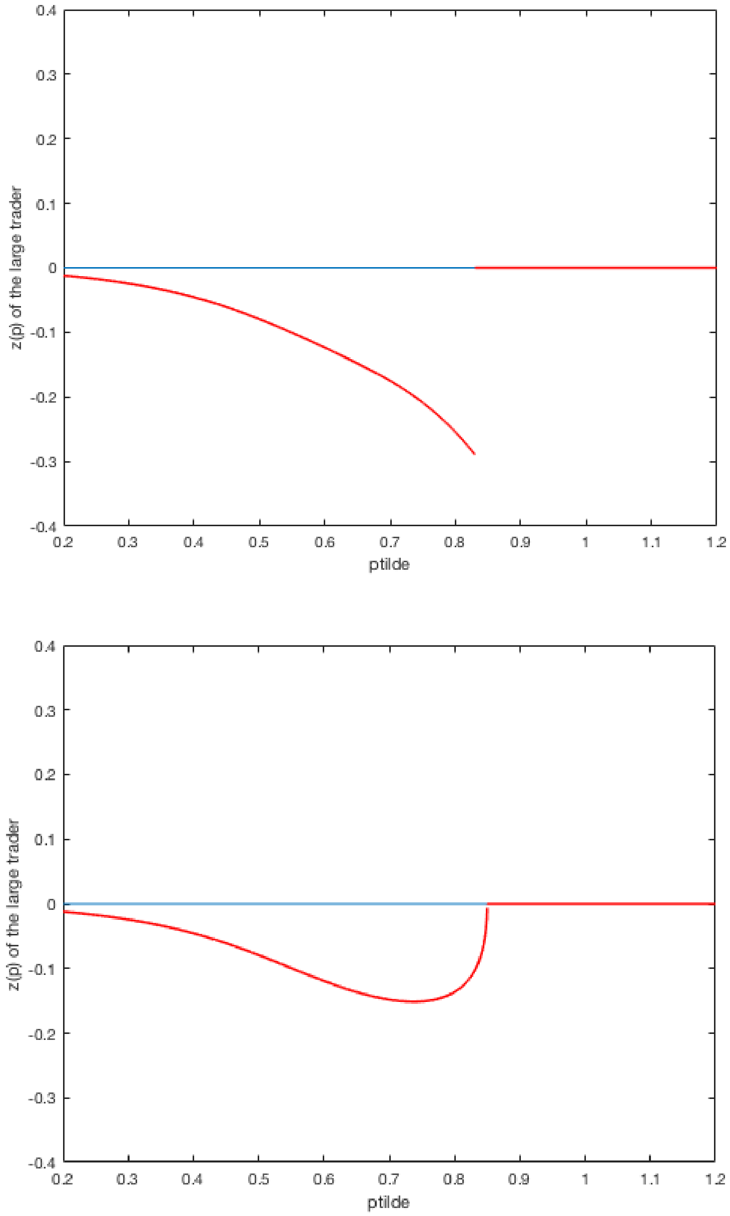

Figure 2.4: The Large Trader's Bidding Function

Parameters: $\sigma_x = 0.5$, $\sigma_\varepsilon = 1$, $\omega = 1$, $\gamma = 2$, $t = 0.5$, $\eta = 1$, $\lambda = 0.7$, $p_1 = 0.85$, $p_2 = 0.7011$.

On the left graph, $\tilde{p}^* = p_1 = 0.85$; on the right graph, $\tilde{p}^* = 0.83 < p_1$.

2.5 "One Large Trader" Model and the Asset Market is Transparent

To draw policy implication regarding the transparency of the asset market, in this section we assume the asset market is transparent in the sense that, the large trader's demand

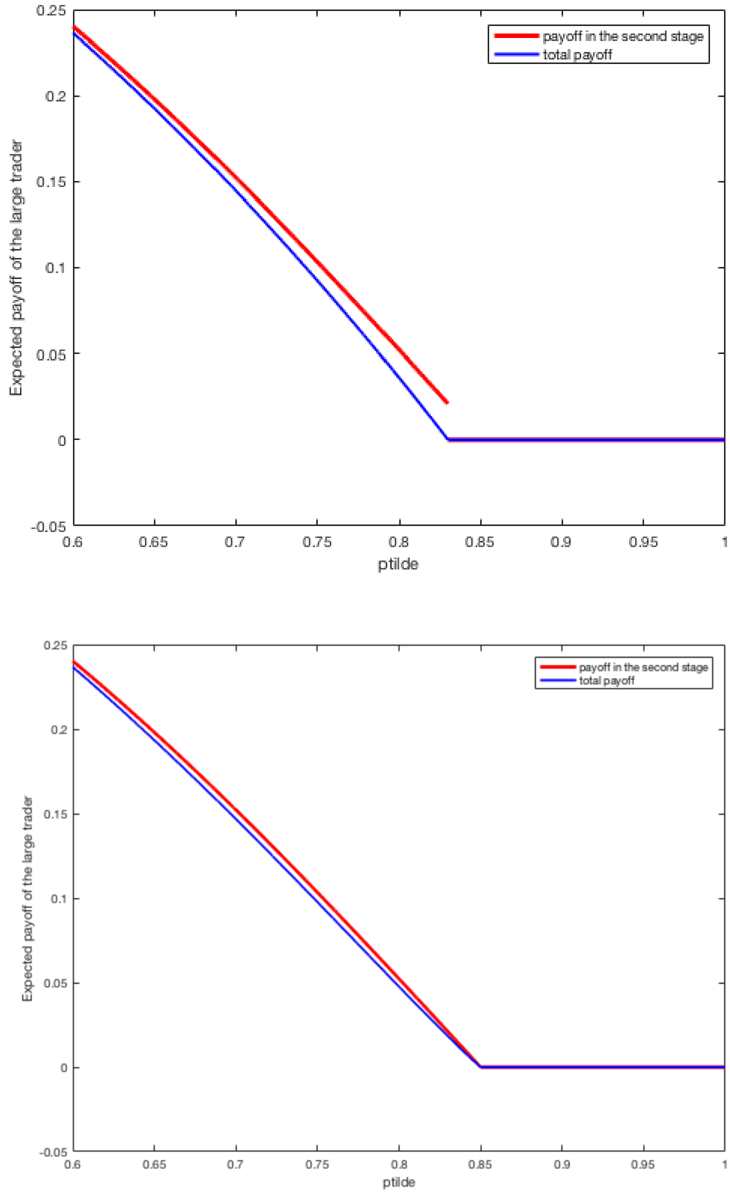

Figure 2.5: The Large Trader's Expected Payoff in the Second-stage Currency-attack Game and Total Expected Payoff

Parameters: $\sigma_x = 0.5$, $\sigma_\varepsilon = 1$, $\omega = 1$, $\gamma = 2$, $t = 0.5$, $\eta = 1$, $\lambda = 0.7$, $p_1 = 0.85$, $p_2 = 0.7011$.

On the left graph, $\tilde{p}^* = p_1 = 0.85$; on the right graph, $\tilde{p}^* = 0.83 < p_1$.

schedule in the asset market is observable for small traders.

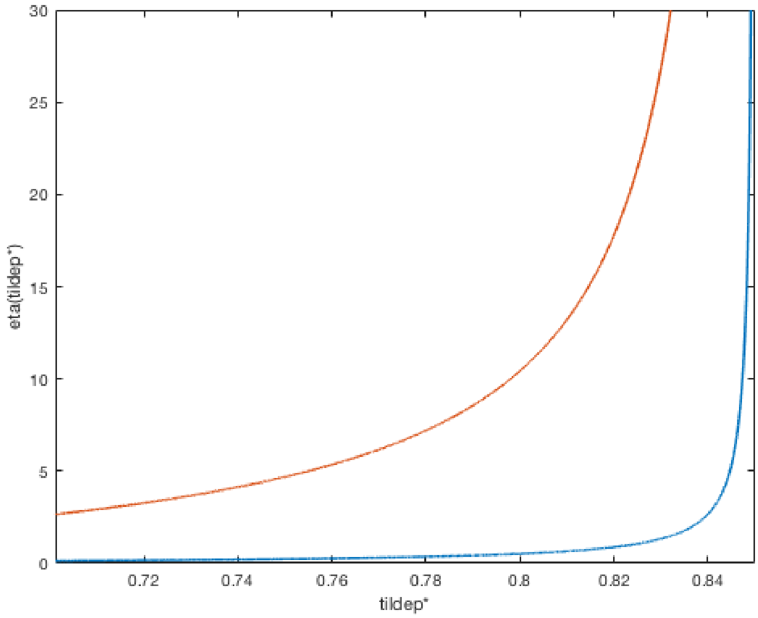

Figure 2.6: $\underline{\eta}(\tilde{p}^*)$ and $\bar{\eta}(\tilde{p}^*)$

Parameters: $\sigma_x = 0.5, \sigma_\varepsilon = 1, \omega = 1, \gamma = 2, t = 0.5, \lambda = 0.7, \tilde{p}_1 = 0.85, \tilde{p}_2 = 0.7011$.

2.5.1 A Continuum of Equilibrium

Definition 5. *Equilibrium is a set of functions*

$$\Big\{p(\theta,\varepsilon,z), k(x,p,z), K(\theta,p,z), z(p), a(x,p,z), a_s(z,p), A(\theta,p,z)\Big\},$$

such that

i. Utility Maximization:

k solves the maximization problem of small trader,

$$k(x,p,z) \in \arg\max_{k\in R} E(u(w_0 + (\theta - p)k)|x,p,z),$$

and

$$K(\theta,p,z) = E(k(x,p,z)|\theta,p,z)\omega;$$

a solves the maximization problem of small trader,

$$a(x,p,z) \in \arg\max_{a\in\{0,1\}} E(u(a, A(\theta,p,z),\theta)|x,p,z);$$

$(z(p), a_s(z,p))$ solves the maximization problem of the large trader,

$$(z(p), a_s(z,p)) \in \arg\max_{z\in R, a\in\{0,1\}} E_s((\theta - p)z\eta + u(a_s, A(\theta,p,z),\theta)|p),$$

where η normalizes the large trader's payoff in the asset market relative to the payoff to a successful attack in the currency-attack game.

$$A(\theta, p, z) = (1-\lambda)E(a(x, p, z)|p, \theta, z) + \lambda a_s(p, z),$$

where

$$u(a, A, \theta) = a(1_{A>\theta} - t),$$

$$u(a_s, A, \theta) = a_s(1_{A>\theta} - t).$$

ii. Market clearing in the first-stage asset market:

$$K(\theta, p, z) + z = K^s(\varepsilon) = \sigma_\varepsilon \varepsilon.$$

iii. Sequential Rationality:

For any $z \neq z(p)$, there exists some belief of trader to support his $a(x, p, z)$ or $a_s(p, z)$ as its optimal strategy,

$\forall z \neq z(p), \exists A \in [0, 1+\lambda]$, *such that*

$$a(x, p, z) \in \arg\max_{a \in \{0,1\}} E(u(a, A, \theta)|x, p, z);$$

$\forall z \neq z(p), \exists A \in [\lambda a_s, (1-\lambda) + \lambda a_s]$, *such that*

$$a_s(p, z) \in \arg\max_{a_s \in \{0,1\}} E(u(a_s, A, \theta)|p, z).$$

The characterization of equilibrium of small traders in the first-stage asset market is the same with "Two Large Traders" model above. The price function is

$$p = \theta - \frac{\gamma\sigma_\varepsilon}{\omega\alpha_x}\varepsilon + \frac{z\gamma}{\omega\alpha}.$$

Define $\tilde{p} := p - \frac{\gamma z(p)}{\omega\alpha}$, which is the de facto public signal about θ used by small traders and the large trader in the second-stage currency-attack game. Replacing p with $\tilde{p}$ in equations (2.1)(2.2), we can solve small trader's strategy $\bar{x}^*(\tilde{p})$ and $\underline{x}^*(\tilde{p})$ in the currency-attack game.

Proposition 15. *We have a continuum of equilibrium. For any $p^* \in [p_2, p_1]$, we can specify an equilibrium where,*

i. the large trader doesn't participate in the asset market, i.e., $z = 0$; $p = \theta - \frac{\gamma\sigma_\varepsilon}{\omega\alpha_x}\varepsilon$; small trader's demand function in the asset market is

$$k(x, p, z) = (\alpha_x x - \alpha_x p - \frac{\gamma\alpha_p}{\omega\alpha}z)\frac{1}{\gamma};$$

ii. the large trader attacks the currency regime if and only if $\tilde{p} \leq p^*$;

iii. if $z = 0$ (on the equilibrium path), small trader attacks the currency regime if and only if $x \leq \bar{x}^*(\tilde{p})$ when $\tilde{p} \leq p^*$, or $x \leq \underline{x}^*(\tilde{p})$ when $\tilde{p} > p^*$;

iv. if $z \neq 0$ (off the equilibrium path), the large trader and small trader's strategies satisfy sequential rationality.

2.5.2 Equilibrium Refinement Using Forward Induction

Our refinement of equilibrium adopts the infinite rounds of elimination of dominated strategies, which incorporates the idea of forward induction. The detailed procedure is shown in the Appendix 4.2.1.

Proposition 16. *Given $k(x, p, z) = (\alpha_x x - \alpha_x p - \frac{\gamma\alpha_p}{\omega\alpha}z)\frac{1}{\gamma}$, the equilibria that survives the infinite rounds of elimination of dominated strategies are,*

$$(z(p), a_s(z, p)) = \begin{cases} (0, 1) \ \textit{if } \tilde{p} < p_1; \\ (0, 0) \ \textit{if } \tilde{p} > p_1; \\ (0, 1) \ \textit{and } (0, 0) \ \textit{if } \tilde{p} = p_1. \end{cases}$$

The equilibrium strategy of small traders in the second-stage currency-attack game

$$(x^*(z = 0, \tilde{p}), x^*(z \neq 0, \tilde{p})) = \begin{cases} (\bar{x}^*(\tilde{p}), \bar{x}^*(\tilde{p})) \ \textit{if } \tilde{p} < p_1; \\ (\underline{x}^*(\tilde{p}), \mathbf{x}(\tilde{p})) \ \textit{if } \tilde{p} > p_1; \\ (\mathbf{x}(\tilde{p}), \bar{x}^*(\tilde{p})) \ \textit{if } \tilde{p} = p_1. \end{cases}$$

where $\mathbf{x}(\tilde{p})$ denotes non-singleton strategy set.

Except for $\tilde{p} = p_1$, this refinement leads to the same outcome with that of the most aggressive equilibrium $p^* = p_1$ among the continuum of equilibrium in Proposition 15. The large trader can easily coordinate an aggressive attack by taking a non-trivial position in the asset market. The public understand that the large trader could have easily coordinated an aggressive attack by taking a non-trivial position in the asset market, and hence an aggressive attack is coordinated even when the large trader does not do so.It highlights how a transparent asset market, as the large trader's coordination device for aggressive currency attack, could jeopardize the currency regime.

2.5.3 Policy Implication

Transparency in the Asset Market

The mechanism found in our paper is in contrast with the traditional wisdom that the complete transparency helps stabilize the currency regime.

Proposition 13 shows it has multiple equilibrium when the asset market is not transparent. Proposition 16 shows if the asset market is transparent, a natural equilibrium refinement that incorporates forward induction reasoning selects the equilibrium where all the traders behave most aggressively in the currency-attack game. Regarding the case with non-transparent asset market as the benchmark, a transparent asset market, as large traders' coordination device for aggressive currency attack, could jeopardize the currency regime.

Transparency in the Currency-attack game

If the large trader's position in the currency-attack game is transparent, then it is equivalent to that the large trader moves first in the currency-attack game and second-stage small traders move afterwards. In our model, the large trader will attack the currency regime if and only if $\tilde{p} \leq \tilde{p}_1$. Small traders will attack if and only if $x_j \leq \bar{x}^*(\tilde{p})$ if $\tilde{p} \leq \tilde{p}_1$, or $x_j \leq \underline{x}^*(\tilde{p})$ if $\tilde{p} > \tilde{p}_1$. The transparency of the currency-attack game removes the large trader's strategic uncertainty for small traders. The equilibrium outcome is identical to that in the most aggressive equilibrium.

From this perspective, the opacity of large traders' position in currency market could vitalize timid equilibrium, and hence do good to the stability of currency regime.

2.6 Conclusion

We use a stylized model to study the large trader's information manipulation and its implication for asset prices during currency crises. To isolate the forces and effects of the information manipulation, the large trader is assumed to be without private information. Our model shows the large trader has the incentive to manipulate the asset price in favor of its currency attack. It delivers the result that in all the equilibrium the large trader will manipulate the asset price to be lower than the price in the case that its demand schedule in the asset market is observable; the large trader's shorting the asset market and attacking the currency regime are concurrent; the large trader's short position in the asset market is most significant when the public signal is in the intermediate range.

The force conveyed in this paper is likely to be at play in other scenarios, such as the market manipulation in 2008 and the manipulation of the deficit and debt statistics by the Greek government in 2000s. Our results also imply that the uninformed manipulator is trapped by its manipulation power.

Finally, we show that if the asset market is transparent, a natural equilibrium refinement that incorporates forward induction reasoning selects the equilibrium where all the traders behave most aggressively in the currency-attack game. It highlights the mechanism how the transparent asset market as large traders' coordination device for aggressive currency attack, could jeopardize the currency regime. That is in contrast with the traditional wisdom that transparency helps stabilize the currency regime.

CHAPTER 3

Determinants of Currency Composition of Reserves: a Portfolio Theory Approach with an Application to RMB

3.1 Introduction

This paper applies a portfolio theory approach to analyze the determinants of the currency composition of foreign exchange (FX) reserves. Even though the need for reserves varies by country, the majority of central banks manage their reserves in a similar approach with common features including:

- Central banks are in general highly risk-averse, with the bulk of their reserves invested in safe assets.[1])
- Central banks often follow a portfolio optimization strategy ([PC03] and [PC05]; [BCC04]) with the mean-variance portfolio diversification approach being a popular one ([Red03]; [De 03]; and [Naa03]).[2]
- "Safety, liquidity, and profitability" are generally accepted as the objectives of reserve management ([Nug00]; and [Fun01]) with safety and liquidity being the primary goals ([BGH08]).
- Central banks often create a "liquidity tranche" portfolio and a "investment portfolio". The liquidity tranche is designed to finance the day-to-day FX needs, fa-

[1]For example, in the case of Canada, the list of eligible reserves currencies is limited to US dollars, euros, Japanese yen, and UK pounds; and the list of eligible asset classes is similarly narrow and limited to fixed-income securities ([MW15]

[2]Central banks have followed the broad trend of the asset management sector of the financial industry, borrowing and putting in place very much the same kind of processes and tools ([BGH08]).

cilitating trade and financial flows.[3] Its asset portfolio often includes exclusively T-bills and time deposits. The "investment portfolio" pursues the highest return subject to risk constraints.[4] It is especially relevant for countries with large reserve holdings that can afford to invest with longer-term growth objectives.[5] Two-thirds of the central banks responding to a BIS survey had established two (or more) separate tranches ([BGH08]). The size of each tranche differs by central bank. In some cases, less than 10 percent of reserves are allocated to liquidity tranche, while in others, over half of reserves are held in this tranche.

- The currency compositions of imports invoicing and short-term external debt could have a significant effect on the currency composition of FX reserves. If reserves are considered mainly as providing an insurance (a "hedge") against the loss of accesss to foreign goods and services, the currency composition of import basket is relevant for the currency composition of reserves. If reserves are seen as primarily hedging against the loss of the access to international financial markets, the currency composition of external liabilities would be more relevant ([BGH08]).[6] Some well-known rules of thumb, such as the ratio of reserves to imports or the ratio of reserves to short-term external liabilities, have been used to assess the adequate level of reserves.

- A shift towards using domestic currency as numeraire (i.e., to serve as a unit of account) may well have taken place ([BGH08], [McC08]; [Rik06]; and [Chi06]).

Incorporating these features into consideration, we introduce a central bank's reserve

[3]For example, in the case of Israel, "About three quarters of the reserve portfolio is invested in very liquid assets. The other quarter is invested in assets with slightly lower liquidity. The ratio is set in relation to the level of reserves and an assessment of the possible need for liquidity."([BI01]).

[4]For example, in the case of Chile, "The diversification portfolio is managed on the basis of a risk budget. This risk budget controls exchange rate risk, interest risk and, partly, credit risk."

[5]For example, the investment portfolio of Hong Kong SAR is invested to preserve the Fund's value for future generations ([BI01]).

[6]As noted in [BI01], in the case of Columbia, "The authorities are now also considering bringing the currency composition of reserves closer in line with that of the stock of short-term external debt"; in the case of Israel, "The benchmark comprises pre-set weights of several currencies, determined more or less in accordance with the currency composition of imports and debt service expenditure in the coming year"; and in the case of Korea, "currency composition is based on the currency of external debt, current payments and market depth and size of reserve assets."

portfolio choice model.

Our approach adopts the classical mean-variance framework for the investment tranche; while the asset-liability framework is adopted for the liquidity tranche. As pointed out by [ST90], the asset-liability framework is appropriate in cases where the types of liabilities are very different (such as FX obligations arising from imports and debt payments).[7] As shown later, the asset-liability framework can deliver a closed-form solution, which makes parameter estimations convenient. Our structural model enables us to quantify the importance of various factors in influencing a central bank's decision.

Using the IMF Currency Composition of Official Foreign (COFER) data, we can quantify the roles of the currency compositions of imports invoicing and short-term external debt, and risk and returns of reserve currencies in determining the currency composition of FX reserves. We can also estimate the likely paths of RMB's share in FX reserves under different scenarios.

Given the absence of structural models in the literature, a key contribution of our paper is to quantify the importance of various factors in determining the currency composition of FX reserves by using a structural model that explicitly models central banks' multi-tranche optimization problem, in particular, at the aggregate level of central banks. Prior to our work, the importance of these factors in determining the currency composition at the aggregate level of central banks was recognized largely qualitatively, and was estimated by some using reduced-form approaches. Our structural approach complements them and has the advantage of identifying the mechanisms that determine the outcomes of currency composition and relating them to central banks' preferences and other factors. In addition, the structural model can conduct counter-factual analyses and scenario-based forecasts.

The remainder of the paper is organized as follows. Section 3.2 reviews the literature. Section 3.3 introduces the stylized patterns of reserves and reserve management, and presents an analysis based on a reduced-form regression. Section 3.4 describes the data. Section 3.5 proposes our portfolio choice model for central banks. Section 3.6 illustrates our estimation strategy and presents results. Section 3.7 analyzes the paths of the share

[7]The original Markowitz model ([Mar52] is a special case of the asset-liability model.

of RMB in FX reserves under different scenarios.[8] Section 3.8 concludes.

3.2 Literature Review

3.2.1 Theory

There is a set of literature that focuses on the modeling of the internationalization of one currency. [Kru84] shows how there can be multiple equilibrium in the use of an international currency. [MKM93] develops a theory of random matching games. [Rey01] shows that the possibility of multiple equilibrium in the internationalization of currencies is determined by network externalities and the pattern of international trade. [GRG10] presents a model of the special "exorbitant privilege" role of the US dollar in the international financial system.

As reserve currency fulfills three roles–an international store of value, a unit of account, and a medium of exchange ([Kru84] and [Fra92]–the literature on safe asset shortages and rollover risk is relevant. [HKM19] reviews this set of literature and links the determination of reserve asset status to the relative fundamentals and relative debt sizes by modeling two countries that issue sovereign bonds to satisfy reserve asset demands from investors.

3.2.2 Empirical Evidence

[Eic98] applies a reduced-form analysis based on the annual aggregate-level FX reserve currency composition data (1971-1995). Its results indicate that a reserve-currency country's shares of global GDP and global trade have significant positive effect on that currency's share in global FX reserves.

[CF07] also uses the aggregate-level data (1973-1998) on the shares of seven main currencies in official reserve holdings to investigate the determinants of the currency composition of international reserves. Their main finding is that the lagged depreciation rate and inflation (or exchange rate volatility) have negative and significant influence

[8]There are many types of reserve buyers, including central banks, private banks, and investment managers. However due to the importance of central banks as reserve buyers, "shares in central banks' foreign exchange reserve holdings are the most important measure of international currency status as well as the most easily measured." ([Fra12])

on the share, while income having a significantly positive influence.[9] They also point out that to attain international currency status, the currency issuing country's financial markets must be not only open and free of control but also deep and well developed.

[DLM89], and [EM00] use confidential IMF data on the country-level currency composition of reserve holdings. Both find that currency pegs, the direction of trade, and the currency of foreign debt are significant and robust determinants of the currency composition of reserve holdings; and their importance is stable across time. [DFG03] and [DFG05] also point out that trade links and currency pegs are the key reasons behind the East Asian and Latin American central banks' unwillingness to follow a pure textbook diversification strategy.[10]

[PPS06] proposes a theoretically grounded and simple mean-variance framework and modifies it to incorporate the specific needs of monetary authorities to hold a sizable portion of their holdings in the currencies of their external debt and the currencies of their main trading partners. Their results indicate that the optimum portfolios show a much lower weight for the euro than is observed. [ZCX12] assumes central banks have two sub-portfolios or tranches; one is with higher risk aversion and fulfills the safety and liquidity objectives and the other is oriented with lower risk aversion and fulfills the profitability objective. [FL04] presents a strategic asset allocation framework and shows that with their assumption of the model's parameters, a typical central bank can significantly increase the efficiency of its portfolio by relaxing the constraints.

With respect to the estimates of RMB's share as reserve currency, [CP10] uses a post-euro data set (1999-2006), and infers from the estimates that RMB could quickly attain the same international currency status as the yen and pound, assuming that China achieved full financial market development. [SK13] forecasts the internalization of RMB

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[9] They also find the shares of major currencies in global reserve holdings are very persistent (the coefficient on the lagged dependent variable is between 0.85 and 0.96). [FO05] builds an open economy model with monopolistic competition among firms that generates high inertia in the currency invoicing of exports. This provides an explanation for the dollar's dominant and stable role in invoicing international trade transactions. Since trade patterns affect the composition of FX reserves, this theory provides one explanation for the high persistence of the dollar in reserve holdings. As our model has taken the trade invoicing into consideration, we do not endogenize the inertia effect through other channel.

[10] The results in [PPS06] suggest that even if the East Asian countries increase their trade with the euro zone and issue euro-denominated securities, as long as they peg their currencies to the dollar, it is very unlikely that they will massively diversify away from the dollar.

from the perspective of reference currency. They show that RMB has increasingly become a reference currency due to trade integration. They forecast that a more global RMB bloc would emerge by the mid-2030s if trade were the sole driver.

3.2.3 Survey and Case Studies

Based on information from central bank asset managers as well as survey data on their reserve policies, Royal Bank of Scotland and the European Central Bank ([PC03] and [PC05]; and [BCC04]) note that central banks do follow a portfolio optimization strategy. The reviews by [Red03] and [De 03] of the asset management practices of the Indian and the Canadian central banks suggest that these institutions pursue mean-variance portfolio diversification policies in their main international reserve holdings. This is further emphasized in [Naa03]'s overview of developing countries' reserves management, which also presents evidence that constraints associated with trade, debt composition, and the currency peg are particularly important in deciding currency composition.

[BGH08] documents the reserve management practices of central banks. Their discussion relies on a survey of central banks and monetary authorities representing in total about 80 percent of global FX reserves at end-2006. They find safety and liquidity are still universally agreed to be the primary goals; at the same time, the weight on the return objectives has generally increased over time; while these trends are common to all central banks, practices still differ considerably, reflecting country-specific circumstances.

3.3 Stylized Patterns and Evidence from a Reduced-form Regression

3.3.1 Currency Compositions of FX Reserves, External Debt, and Imports Invoicing

Figure 3.1 shows the main emerging and developing countries' aggregate FX reserves in main reserve currencies (EUR, JPY, GBP, and USD), imports and short-term external debt denominated in these reserve currencies between 2006 Q1 and 2014 Q4.[11] It shows

[11] The data source and calculation are shown in Section 3.3.

a rapid accumulation of reserves.

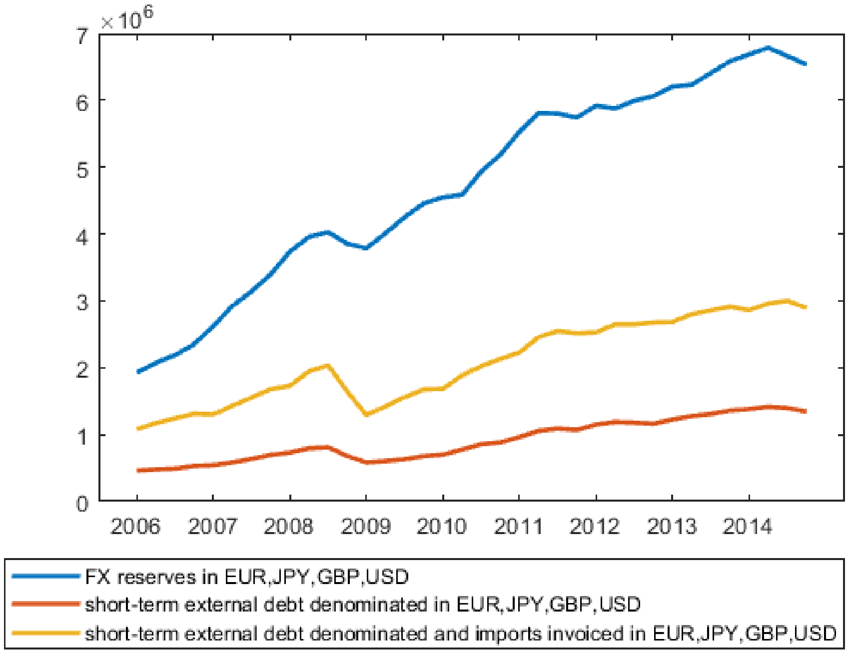

Figure 3.1: FX Reserves, Imports Invoicing, and Short-term External Debt Denominated in EUR, JPY, GBP, and USD (2006 Q1 - 2014 Q4, in mln USD)

Sources: Federal Reserve Bank of St. Louis; [Gop15]; Haver; IMF COFER and DOTS; UNCTAD; World Bank International Debt Statistics; and authors' calculations.

The shares of euro and dollar in import invoicing and external debt are moving in line with their shares in FX reserves (Figure 3.2). The shares of dollar in FX reserves and external debt have increased. Cross-sectional data also shows that the shares of of euro and dollar in external debt are in line with their shares in FX reserves (Figure 3.3).

3.3.2 Domestic Currency as Numeraire

When central banks measure the risk and returns in reserve management, different numeraire choices would generally imply different optimal allocations. For example, if a domestic currency varies less against the dollar than other major currencies, a reserve portfolio with a substantial dollar share poses less risk when returns are measured in that domestic currency. The central banks in Australia ([Val12]) and Chile ([Chi12]) are

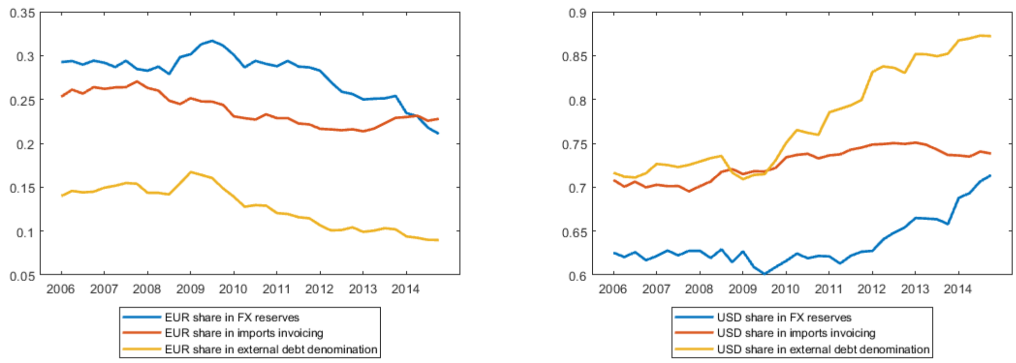

Figure 3.2: The Shares of EUR and USD in FX Reserves, Imports Invoicing, and Short-term External Debt (2006 Q1 - 2014 Q4)

Sources: [Gop15]; Haver; IMF COFER and DOTS; UNCTAD; World Bank International Debt Statistics; and authors' calculations.

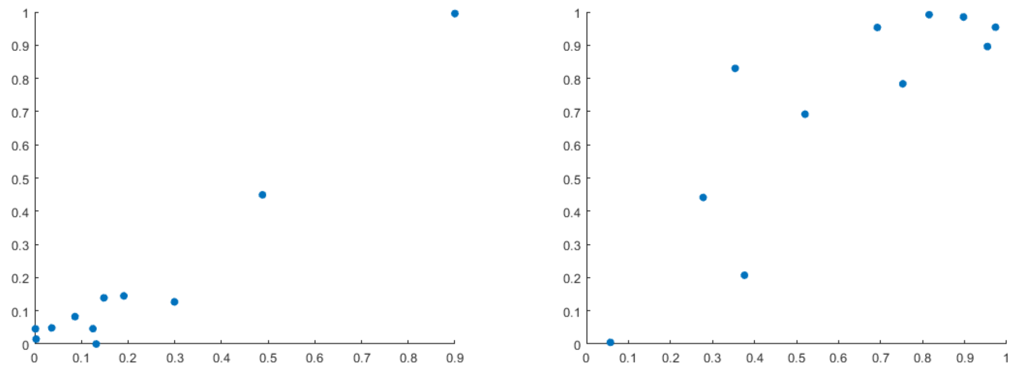

Figure 3.3: The Shares of EUR (left) and USD (right) in FX Reserves versus in External Debt (2016 Q4)

Sources: Haver; IMF COFER; World Bank International Debt Statistics; and authors' calculations.

examples of using domestic currency as numeraire.

[MC14], and [MIC15] provide evidence consistent with the use of domestic currency as numeraire. They estimate the currency movement for 25 economies, and find cross-sectional evidence that the share of the dollar in reserves is higher where the domestic currency varies less against the dollar than other major currencies. The relationship still holds after they exclude currencies with currency peg and currencies that the IMF characterizes as "crawl-like" or "other managed arrangement", which indicates the relationship does not depend on economies where the currency is heavily managed.

In order to quantify each country's currency co-movement with main reserve currencies, we adopt the method developed by [FW94] and [FW07].

$$\Delta \ln \frac{X_t}{CHF_t} = \rho_1 \Delta \ln \frac{EUR_t}{CHF_t} + \rho_2 \Delta \ln \frac{JPY_t}{CHF_t} + \rho_3 \Delta \ln \frac{GBP_t}{CHF_t} + \rho_4 \Delta \ln \frac{USD_t}{CHF_t} + \alpha + \varepsilon_t,$$

where $\frac{X_t}{CHF_t}$ is each country's exchange rate versus Swiss Franc. We name ρ_1-ρ_4 the weights of euro zone, yen zone, pound zone, and dollar zone.

Following [MC14], we use a 10-year sample (2007 to September 2017) to estimate ρ_1 to ρ_4 for each country which releases its currency composition to COFER.[12] The results are consistent with the use of domestic currency as numeraire (Figure 3.4).

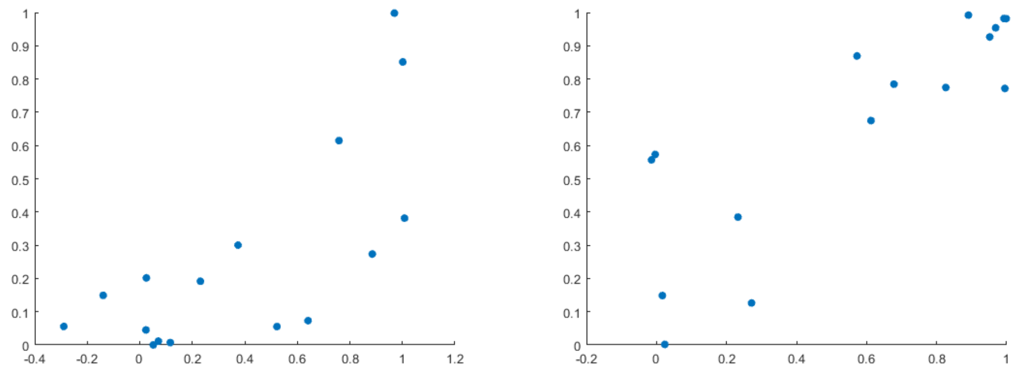

Figure 3.4: The Share of EUR in FX vs Euro Zone Weight (left); and the Share of USD in FX vs Dollar Zone Weight (right) (2017 Q4)

Sources: Bloomberg Finance L.P.; IMF COFER; and authors' calculations.

3.3.3 Exchange Rate Risk and Interest Rate Risk

Our focus is on the exchange rate risk, as the exchange rate movement is much more volatile than the interest rate movement. It is consistent with the well-recognized stability of the government bond yield (Figures 3.5 and 3.6). For example, in the case of Australia,"The most significant of these risks is an exposure to fluctuations in the value of the Australian dollar against the currencies in which reserves are held" ([Val12]). As

[12]Euro zone countries (Belgium, Estonia, Finland, Germany, Ireland, Netherlands, and Portugal) are excluded in our analysis, as their $\rho_1 = 1$ by definition. Moldova and West Bank and Gaza are also excluded due to data limits.

shown later, our framework can be extended to incorporate the interest risk and other types of risks.

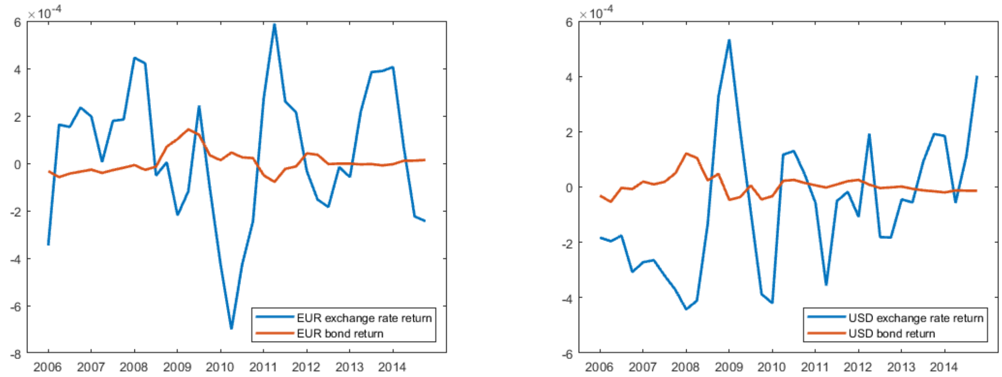

Figure 3.5: Within-one-year Means of EUR, and USD Exchange Rate Returns and 2Y Government Bond Returns Averaged for Brazil, China, India, Russia, and Saudi Arabia (2006 Q1 - 2014 Q4)

Sources: Bloomberg Finance L.P.; and authors' calculations.

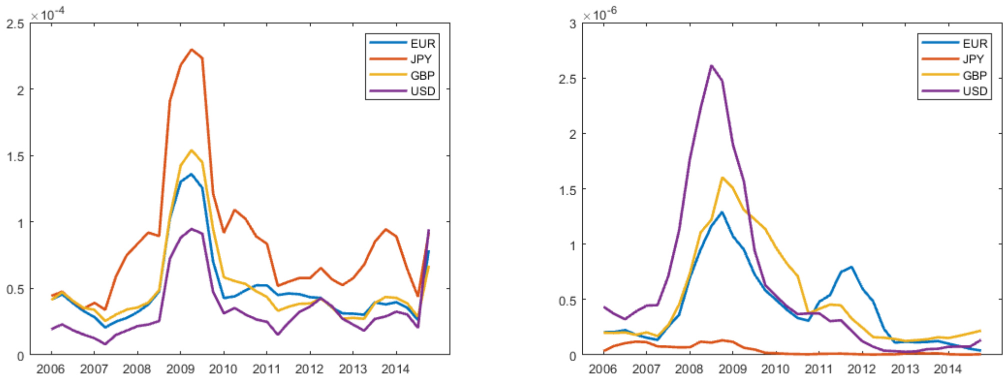

Figure 3.6: Within-one-year Variance of Exchange Rate Returns (left) and 2Y Government Bond Returns (right) Averaged for Brazil, China, India, Russia, and Saudi Arabia (2006 Q1 - 2014 Q4)

Sources: Bloomberg Finance L.P.; and authors' calculations.

3.3.4 Evidence from a Reduced-form Regression

A reduced-form analysis is used to analyze the effect of the currency compositions of imports invoicing and short-term external debt, and risk and returns of exchange rates on the currency composition of FX reserves. The regression model can be presented as

$$x_{it} = c_i + \beta_1 \text{imports invoicing}_{it} + \beta_2 \text{average return}_{it} + \beta_3 \text{return volatility}_{it} + \varepsilon_{it}.$$

The fixed-effect estimators are the OLS estimators for

$$x_{it} - \bar{x}_i = \beta_1(\text{imports invoicing}_{it} - \overline{\text{imports invoicing}_{it}}) + \beta_2(\text{average return}_{it} - \overline{\text{average return}_i})$$

$$+\beta_3(\text{return volatility}_{it} - \overline{\text{return volatility}_i}) + (\varepsilon_{it} - \bar{\varepsilon}_i),$$

where x_{it} is the share of reserve currency i (either EUR, JPY, GBP, or USD) in period t among our sample of emerging and developing countries' FX reserves; imports invoicing$_{it}$ is the share of currency i among these countries' imports invoicing in period t; average return$_{it}$ is currency i's average return rates using SDR as numeraire in the past 7 years up to each quarter; and return volatility$_{it}$ is currency i's volatility of return rates using SDR as numeraire in the past 7 years up to each quarter. The results are presented in column (1) of Table 3.1.

In column (2), we use the share of reserve currency i of these countries' short-term external debt. In column (3), we use the weighted average of the currency shares in imports invoicing and short-term external debt. The aggregate imports invoiced in the four reserve currencies is the weight for the former and the aggregate short-term external debt denominated in these currencies is the weight for the latter. In column (4), the 2-year government bond bid-ask spread is introduced as the control variable.

The results show that the currency compositions of imports invoicing and short-term external debt have significant effect on the currency composition of FX reserves; the return rates also have significant effect, consistent with the existence of the investment tranche; and the effect of the return volatility is not statistically significant, consistent with the results in [CP10].

The effect of the government bond spread is not statistically significant. We believe that, for these four main reserve currencies, the government bonds of their issuers are liquid enough so that their liquidity is not a major concern for reserve buyers. However, as discussed in Section 3.7.1, a liquid government bond market could be one necessary condition for central banks to hold these bonds.

Table 3.1: Results of the Reduced-form Regression

	(1)	(2)	(3)	(4)
imports invoicing	0.622***			
	(4.90)			
ST external debt denomination		0.413***		
		(11.20)		
imports invoicing and ST external debt denomination			0.549***	0.550***
			(9.96)	(9.98)
average return	47.60**	97.10***	64.26***	69.51***
	(2.15)	(6.06)	(3.76)	(3.87)
return volatility	-3.602	1.383	-0.318	-1.714
	(-1.46)	(0.69)	(-0.15)	(-0.67)
government bond spread				-9.588
				(-0.94)
const	0.117***	0.154***	0.115***	0.123***
	(3.53)	(11.48)	(6.35)	(6.12)
N	144	144	144	144
within R-square	0.2459	0.5374	0.4859	0.4892

t statistics in parentheses.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

3.4 Data

Our main sample includes 22 emerging and developing countries.[13] The selection criteria is that they either had FX reserves exceeding USD10bn in 2017, or are included in the imports invoicing dataset in [Gop15]. As the aggregate FX reserves of these 22 countries accounted for 88 percent of the total FX reserve holdings of the emerging and developing

[13]They are Argentina, Brazil, Bulgaria, China, Columbia, Hungary, India, Indonesia, Jordan, Malaysia, Mexico, Morocco, Pakistan, Peru, Philippines, Poland, Romania, Russia, Saudi Arabia, Thailand, Turkey, and Ukraine.

economies in 2015 Q1, we use the currency composition of the emerging and developing countries' aggregate FX reserves reported by COFER[14] to approximate the currency composition of these 22 countries' aggregate FX reserves. The time period covered in our estimation is from 2006 Q1 to 2014 Q4, as the currency composition of emerging and developing countries' FX reserves became unavailable starting from 2015 Q1.

The data on the average currency composition of imports invoicing between 1999 and 2015 is from [Gop15]. We use that average to calculate the imports invoicing in each currency for each year due to data limits.[15] For countries included in the dataset but with missing share for some reserve currency, and for those not included in the dataset, we approximate the currency composition of their imports invoicing as follows:

- If the country is included in the dataset but the share of some reserve currency is missing, we approximate the share using that currency's share in the imports invoicing of the countries with available data.
- If the country is not included in the dataset, we approximate each reserve currency's share using that currency's share in the imports invoicing of the countries with available data.

After the approximation, the sum of each country's imports invoiced in that reserve currency becomes the aggregate imports of all these 22 countries invoiced in that reserve currency.

Approximation is also applied for the currency composition of short-term external debt. The data on country-level short-term external debt is sourced from Haver. As there is almost no data on the currency composition of short-term external debt, we use the currency composition of total external debt or public and publicly-guaranteed (PPG) debt from World Bank International Debt Statistics as an approximation. In cases where PPG debt is unavailable for one country, we use the short-term external debt-weighted average of other countries' currency shares as the approximation for that country. Next we calculate each country's aggregate short-term external debt denominated in reserve

[14] COFER data for individual countries are strictly confidential, except for several countries.

[15] The imports of goods data is from IMF DOTS; and the imports of service data is from UNCTAD.

currencies; and the aggregate short-term external debt of all these 22 countries denominated in each reserve currency.

To investigate the cross-sectional pattern, we use the rather limited quarterly country-level reserve currency composition data from the IMF COFER data.[16] For each country, we calculate the currency share of EUR, JYP, GBP, and USD.

3.5 A Portfolio Choice Model

We have developed a portfolio choice model to analyze the currency allocation decision of central banks. In the model, the aggregate reserve portfolio of a central bank is divided into two tranches, liquidity tranche and investment tranche. The size of the FX obligations, denoted by L, is related to the value of FX used for the payment for imports and short-term external debt, using domestic currency as numeraire. There are other types of liabilities that could potentially call for FX, such as domestic deposits denominated in FX, M2, and other types of external liabilities ([Fun11]). However, in this paper we only consider FX payment for imports and short-term external debt denominated in FX, as they are the most relevant FX-denominated potential drains for many countries. y_i denotes the share of obligations in currency $i \in I$. R_i denotes the growth rate of the value of currency i obligations, which equals to the growth rate of the exchange rate of currency i using domestic currency as numeraire. The value of the obligations one period later is

$$\sum_i Ly_i(1+R_i) = L(1+\sum_i y_i R_i),$$

where A denotes the initial value of reserve assets (the aggregate portfolio). If $A > L$, the size of the liquidity tranche equals to L, the size of the investment tranche is $A - L$. If $A \leq L$, the whole portfolio is the liquidity tranche.

For simplicity, in our benchmark model, we assume the returns of the reserve assets using their denominate currency as numeraire are zero, therefore exchange rates are the only factor that affects the values of reserves assets. Under this assumption, as the assets

[16]The data is available for Australia, Belgium, Brazil, Bulgaria, Chile, Costa Rica, Croatia, Estonia, Finland, Georgia, Germany, Ireland, Israel, Latvia, Mauritius, Moldova, Morocco, Netherlands, Norway, Peru, Portugal, Seychelles, Sweden, Switzerland, Ukraine, Uruguay, West Bank and Gaza.

denominated in the same reserve currency are essentially the same, the central banks' problem becomes a choice of currency allocation. This assumption is consistent with the observation of a much larger FX risk compared to interest rate risk (Figures 3.5 and 3.6). In reality, central banks we have seen do set the shares of reserve currencies as benchmark when managing their FX reserves.[17] This assumption can be relaxed with details shown in Appendix 4.3.1.

The central bank chooses the currency share x_{Li}, x_{Ii} of each currency i for the liquidity tranche and the investment tranche. The value of the liquidity tranche after one period is

$$\sum_i L x_{Li}(1+R_i) = L(1+\sum_i x_{Li}R_i).$$

The value of the investment tranche after one period is

$$\sum_i (A-L)x_{Ii}(1+R_i) = (A-L)(1+\sum_i x_{Ii}R_i).$$

The goal for the liquidity tranche is to maximize the surplus subject to some risk constraints[18]

$$L\sum_i x_{Li}R_i - L\sum_i y_i R_i.$$

It is equivalent to maximize $\frac{L\sum_i x_{Li}R_i - L\sum_i y_i R_i}{L} = \sum_i (x_{Li} - y_i)R_i$. The central bank's optimization problem for the liquidity tranche is

$$\min_{\mathbf{x}_L} \frac{1}{2}(\mathbf{x}_L - \mathbf{y})^{\intercal}\boldsymbol{\Omega}(\mathbf{x}_L - \mathbf{y}) - \lambda_L \mathbf{m}^{\intercal}(\mathbf{x}_L - \mathbf{y}),$$

s.t. $\mathbf{e}^{\intercal}\mathbf{x}_L = 1$.

where $\boldsymbol{\Omega}$ is the covariance matrix of the currency return rates, $\mathbf{m}$ is the expected return rates of currencies. The solution is

$$\mathbf{x}_L = \mathbf{y} + \lambda_L(\boldsymbol{\Omega}^{-1}\mathbf{m} - \frac{\mathbf{e}^{\intercal}\boldsymbol{\Omega}^{-1}\mathbf{m}}{\mathbf{e}^{\intercal}\boldsymbol{\Omega}^{-1}\mathbf{e}}\boldsymbol{\Omega}^{-1}\mathbf{e}).$$

[17] For example, Chile' central bank sets the reference structure of the medium-term liquidity portfolio to be USD 39%, and EUR 36% (Central Bank of Chile, 2012); for Sweden, its benchmark is USD 35%, EUR 35%, GBP 15% and JPY 15%; for UK, its benchmark is USD 40%, EUR 40%, and JPY 20%; for Hong Kong, the benchmark is USD 80%, EUR 15%, JPY 5%; for Colombia, the benchmark is USD 80.4%, EUR 15.3%, JPY 4.2%; and for Botswana, the currency composition of the short-term fund is USD 48.9%, EUR 25.2%, GBP 11.1%, JPY 11.9% ([BI01]).

[18] For example, for Colombia, its currency composition of the benchmark is determined by the expected denomination of the balance of payments outflows...in order to maintain the value of the reserves in terms of expected outflows ([BI01]).

$\mathbf{y}$ is the minimum variance portfolio ($\lambda_L = 0$). If central banks do not have return concern for the liquidity tranche, the currency composition of the liquidity tranche will be set to be equal to the currency composition of the obligations.

The goal for the investment tranche is to maximize the **returns** subject to some risk constraints. It is equivalent to maximize $1 + \sum_i x_{Ii} R_i$. Thus the central bank's optimization problem for the investment tranche is

$$\min_{\mathbf{x}_I} \frac{1}{2}\mathbf{x}_I^{\intercal}\mathbf{\Omega}\mathbf{x}_I - \lambda_I \mathbf{m}^{\intercal}\mathbf{x}_I,$$

s.t. $\mathbf{e}^{\intercal}\mathbf{x}_I = 1$.

The solution is

$$\mathbf{x}_I = \frac{\mathbf{\Omega}^{-1}\mathbf{e}}{\mathbf{e}^{\intercal}\mathbf{\Omega}^{-1}\mathbf{e}} + \lambda_I(\mathbf{\Omega}^{-1}\mathbf{m} - \frac{\mathbf{e}^{\intercal}\mathbf{\Omega}^{-1}\mathbf{m}}{\mathbf{e}^{\intercal}\mathbf{\Omega}^{-1}\mathbf{e}}\mathbf{\Omega}^{-1}\mathbf{e}).$$

If $A > L$, the currency composition of the whole reserve portfolio is

$$\mathbf{x} = \frac{L\mathbf{x}_L + (A-L)\mathbf{x}_I}{A} = \mathbf{x}_I + \frac{L}{A}(\mathbf{x}_L - \mathbf{x}_I)$$

$$= \frac{\mathbf{\Omega}^{-1}\mathbf{e}}{\mathbf{e}^{\intercal}\mathbf{\Omega}^{-1}\mathbf{e}} + \lambda_I(\mathbf{\Omega}^{-1}\mathbf{m} - \frac{\mathbf{e}^{\intercal}\mathbf{\Omega}^{-1}\mathbf{m}}{\mathbf{e}^{\intercal}\mathbf{\Omega}^{-1}\mathbf{e}}\mathbf{\Omega}^{-1}\mathbf{e}) + \frac{L}{A}(\mathbf{y} - \frac{\mathbf{\Omega}^{-1}\mathbf{e}}{\mathbf{e}^{\intercal}\mathbf{\Omega}^{-1}\mathbf{e}} + (\lambda_L - \lambda_I)(\mathbf{\Omega}^{-1}\mathbf{m} - \frac{\mathbf{e}^{\intercal}\mathbf{\Omega}^{-1}\mathbf{m}}{\mathbf{e}^{\intercal}\mathbf{\Omega}^{-1}\mathbf{e}}\mathbf{\Omega}^{-1}\mathbf{e})).$$

If $A \leq L$, the whole portfolio is the liquidity tranche; we assume the central banks only take the maximal size of obligation that can be covered by the reserves, that is A, into consideration, which implies $\mathbf{x} = \mathbf{x}_L$.[19]

In the optimization problems shown above, we assume central banks use their domestic currencies as the numeraire when valuing the investment tranche and the surplus of the liquidity tranche. The choice of numeraire matters for the optimal currency allocation, as the returns and risk of reserve currencies are affected by which currency is chosen as the unit of account. Based on the empirical evidence and case studies, using domestic currency as the numeraire is a reasonable assumption to make. However it does not preclude that some countries might use USD or SDR or a basket of currencies as the numeraire. If the return is not the concern for the liquidity tranche ($\lambda_L = 0$), the

[19]Thus this assumption leads to the result that if the return is not the concern for their liquidity tranche, then for the central banks with inadequate reserves, the currency composition of their reserve portfolio is in accordance with the that of obligations. This result is consistent with many central banks' practices.

currency composition of the liquidity tranche is determined by the currency composition of the obligations; and therefore only the currency composition of the investment tranche is affected by the choice of numeraire.

The FX obligations depend on the imports payment and short-term external debt service,

$$L\mathbf{y} = a\mathbf{T} + b\mathbf{D},$$

where $\mathbf{T}$ are the imports invoicing in foreign currencies, $\mathbf{D}$ are the short-term external debt denominated in foreign currencies. As the foreign currencies in imports invoicing and external debt denomination are dominated by EUR, JPY, GBP, and USD,[20] we only need to look at the import invoicing and short-term external debt denominated in these four main reserve currencies. Our model and estimation can be extended to incorporate more currencies in imports invoicing and short-term external debt denomination.

The aggregate obligation and the size of the liquidity tranche are

$$L = \mathbf{e}^\intercal L\mathbf{y} = \mathbf{e}^\intercal(a\mathbf{T} + b\mathbf{D}),$$
$$\mathbf{y} = \frac{a\mathbf{T} + b\mathbf{D}}{\mathbf{e}^\intercal(a\mathbf{T} + b\mathbf{D})}.$$

In some period, for the central bank with $L < A$, there is investment tranche. These central banks are denoted by $l \in J_1$; for the central bank with $L \geq A$, there is no investment tranche. These central banks are denoted by $l \in J_2$.

$$l \in \begin{cases} J_1 & \text{if } \ \mathbf{e}^\intercal(a\mathbf{T}_l + b\mathbf{D}_l) < A_l; \\ J_2 & \text{if } \ \mathbf{e}^\intercal(a\mathbf{T}_l + b\mathbf{D}_l) \geq A_l. \end{cases}$$

The optimal currency composition of each of central bank $l \in J_1$ are

$$\mathbf{x} = \frac{\mathbf{\Omega}^{-1}\mathbf{e}}{\mathbf{e}^\intercal\mathbf{\Omega}^{-1}\mathbf{e}} + \lambda_I(\mathbf{\Omega}^{-1}\mathbf{m} - \frac{\mathbf{e}^\intercal\mathbf{\Omega}^{-1}\mathbf{m}}{\mathbf{e}^\intercal\mathbf{\Omega}^{-1}\mathbf{e}}\mathbf{\Omega}^{-1}\mathbf{e}) + \frac{1}{A}(a(\mathbf{T} - \frac{\mathbf{\Omega}^{-1}\mathbf{e}}{\mathbf{e}^\intercal\mathbf{\Omega}^{-1}\mathbf{e}}\mathbf{e}^\intercal\mathbf{T}) + b(\mathbf{D} - \frac{\mathbf{\Omega}^{-1}\mathbf{e}}{\mathbf{e}^\intercal\mathbf{\Omega}^{-1}\mathbf{e}}\mathbf{e}^\intercal\mathbf{D}))$$
$$+\frac{1}{A}(\lambda_L - \lambda_I)(\mathbf{\Omega}^{-1}\mathbf{m} - \frac{\mathbf{e}^\intercal\mathbf{\Omega}^{-1}\mathbf{m}}{\mathbf{e}^\intercal\mathbf{\Omega}^{-1}\mathbf{e}}\mathbf{\Omega}^{-1}\mathbf{e})\mathbf{e}^\intercal(a\mathbf{T} + b\mathbf{D}).$$

The optimal currency composition of central bank $l \in J_2$ are,

$$\mathbf{x} = \frac{a\mathbf{T} + b\mathbf{D}}{\mathbf{e}^\intercal(a\mathbf{T} + b\mathbf{D})} + \lambda_L(\mathbf{\Omega}^{-1}\mathbf{m} - \frac{\mathbf{e}^\intercal\mathbf{\Omega}^{-1}\mathbf{m}}{\mathbf{e}^\intercal\mathbf{\Omega}^{-1}\mathbf{e}}\mathbf{\Omega}^{-1}\mathbf{e}).$$

[20] Based on the dataset from [Gop15], almost all the imports are invoiced in EUR, JPY, GBP, USD. In our sample, the share of external debt denominated in EUR, JPY, GBP, USD is nearly 90%. The ratio would be even higher if the external debt denominated in the domestic currency is excluded.

By using the data of reserve size, we can derive the optimal currency composition of FX reserves of all central banks $l \in J_1$ and $l \in J_2$, assuming each central bank has the same λ_L, λ_I, a, b,[21]

$$\mathbf{x}_{aggregate}(\mathbf{A}, \mathbf{\Omega}, \mathbf{m}, \mathbf{T}, \mathbf{D}, a, b, \lambda_I, \lambda_L) := \frac{\sum_{l\in J} A_l \mathbf{x}_l}{\sum_{l\in J} A_l} = \frac{\sum_{l\in J_1} A_l \frac{\mathbf{\Omega}_l^{-1}\mathbf{e}}{\mathbf{e}^\intercal \mathbf{\Omega}_l^{-1}\mathbf{e}}}{\sum_{l\in J} A_l}$$

$$+a\frac{\sum_{l\in J_1}(\mathbf{T}_l - \frac{\mathbf{\Omega}_l^{-1}\mathbf{e}}{\mathbf{e}^\intercal \mathbf{\Omega}_l^{-1}\mathbf{e}}\mathbf{e}^\intercal \mathbf{T}_l)}{\sum_{l\in J} A_l} + b\frac{\sum_{l\in J_1}(\mathbf{D}_l - \frac{\mathbf{\Omega}_l^{-1}\mathbf{e}}{\mathbf{e}^\intercal \mathbf{\Omega}_l^{-1}\mathbf{e}}\mathbf{e}^\intercal \mathbf{D}_l)}{\sum_{l\in J} A_l} + \frac{\sum_{l\in J_2} A_l \frac{a\mathbf{T}_l + b\mathbf{D}_l}{\mathbf{e}^\intercal(a\mathbf{T}_l + b\mathbf{D}_l)}}{\sum_{l\in J} A_l}$$

$$+\lambda_I(\frac{\sum_{l\in J_1} A_l(\mathbf{\Omega}_l^{-1}\mathbf{m}_l - \frac{\mathbf{e}^\intercal\mathbf{\Omega}_l^{-1}\mathbf{m}_l}{\mathbf{e}^\intercal \mathbf{\Omega}_l^{-1}\mathbf{e}}\mathbf{\Omega}_l^{-1}\mathbf{e})}{\sum_{l\in J} A_l} - \frac{\sum_{l\in J_1}(\mathbf{\Omega}_l^{-1}\mathbf{m}_l - \frac{\mathbf{e}^\intercal\mathbf{\Omega}_l^{-1}\mathbf{m}_l}{\mathbf{e}^\intercal \mathbf{\Omega}_l^{-1}\mathbf{e}}\mathbf{\Omega}_l^{-1}\mathbf{e})\mathbf{e}^\intercal(a\mathbf{T}_l + b\mathbf{D}_l)}{\sum_{l\in J} A_l})$$

$$+\lambda_L(\frac{\sum_{l\in J_1}(\mathbf{\Omega}_l^{-1}\mathbf{m}_l - \frac{\mathbf{e}^\intercal\mathbf{\Omega}_l^{-1}\mathbf{m}_l}{\mathbf{e}^\intercal \mathbf{\Omega}_l^{-1}\mathbf{e}}\mathbf{\Omega}_l^{-1}\mathbf{e})\mathbf{e}^\intercal(a\mathbf{T}_l + b\mathbf{D}_l)}{\sum_{l\in J} A_l} + \frac{\sum_{l\in J_2} A_l(\mathbf{\Omega}_l^{-1}\mathbf{m}_l - \frac{\mathbf{e}^\intercal\mathbf{\Omega}_l^{-1}\mathbf{m}_l}{\mathbf{e}^\intercal \mathbf{\Omega}_l^{-1}\mathbf{e}}\mathbf{\Omega}_l^{-1}\mathbf{e})}{\sum_{l\in J} A_l}). \tag{3.1}$$

3.6 Estimation

3.6.1 Results

For each quarter t and central bank l, the covariance matrix $\mathbf{\Omega}_{lt}$ and mean vector $\mathbf{m}_{lt}$ of the reserve currencies can be estimated using the latest seven-year sample up to quarter t. For each central bank l, the choice of the numeraire affects $\mathbf{\Omega}_{lt}$ and $\mathbf{m}_{lt}$ of the reserve currencies. In our estimation we assume each central bank uses its domestic currency as the numeraire. This assumption can be relaxed if more information about central banks' numeraire choices are available.

Define $\mathbf{\Lambda}_t := (\mathbf{A}_t, \mathbf{\Omega}_t, \mathbf{m}_t, \mathbf{T}_t, \mathbf{D}_t)$ and $\mathbf{u}_i(\mathbf{\Lambda}_t, a, b)$ as a 9×1 vector,

$$\mathbf{u}_{i1} = \frac{\sum_{l\in J_1} A_{lt}\frac{(\mathbf{\Omega}_{lt}^{-1}\mathbf{e})_i}{\mathbf{e}^\intercal\mathbf{\Omega}_{lt}^{-1}\mathbf{e}}}{\sum_{l\in J} A_{lt}}, \mathbf{u}_{i2} = \frac{\sum_{l\in J_1}(\mathbf{T}_{lt} - \frac{\mathbf{\Omega}_{lt}^{-1}\mathbf{e}}{\mathbf{e}^\intercal\mathbf{\Omega}_{lt}^{-1}\mathbf{e}}\mathbf{e}^\intercal\mathbf{T}_{lt})_i}{\sum_{l\in J} A_{lt}}, \mathbf{u}_{i3} = \frac{\sum_{l\in J_1}(\mathbf{D}_{lt} - \frac{\mathbf{\Omega}_{lt}^{-1}\mathbf{e}}{\mathbf{e}^\intercal\mathbf{\Omega}_{lt}^{-1}\mathbf{e}}\mathbf{e}^\intercal\mathbf{D}_{lt})_i}{\sum_{l\in J} A_{lt}},$$

$$\mathbf{u}_{i4} = \frac{\sum_{l\in J_2} A_{lt}\frac{(\mathbf{T}_{lt})_i}{\mathbf{e}^\intercal(a\mathbf{T}_{lt} + b\mathbf{D}_{lt})}}{\sum_{l\in J} A_{lt}}, \mathbf{u}_{i5} = \frac{\sum_{l\in J_2} A_{lt}\frac{(\mathbf{D}_{lt})_i}{\mathbf{e}^\intercal(a\mathbf{T}_{lt} + b\mathbf{D}_{lt})}}{\sum_{l\in J} A_{lt}},$$

[21]The assumption of homogeneous λ_I, λ_L, a, b among central banks can be relaxed. Due to a relatively small sample of observations, this assumption is made to reduce the number of parameters we need to estimate.

$$\mathbf{u}_{i6} = \frac{\sum_{l \in J_1} A_{lt}(\boldsymbol{\Omega}_{lt}^{-1}\mathbf{m}_{lt} - \frac{\mathbf{e}^\intercal \boldsymbol{\Omega}_{lt}^{-1}\mathbf{m}_{lt}}{\mathbf{e}^\intercal \boldsymbol{\Omega}_{lt}^{-1}\mathbf{e}}\boldsymbol{\Omega}_{lt}^{-1}\mathbf{e})_i}{\sum_{l \in J} A_{lt}},$$

$$\mathbf{u}_{i7} = \frac{\sum_{l \in J_1} (\boldsymbol{\Omega}_{lt}^{-1}\mathbf{m}_{lt} - \frac{\mathbf{e}^\intercal \boldsymbol{\Omega}_{lt}^{-1}\mathbf{m}_{lt}}{\mathbf{e}^\intercal \boldsymbol{\Omega}_{lt}^{-1}\mathbf{e}}\boldsymbol{\Omega}_{lt}^{-1}\mathbf{e})_i \mathbf{e}^\intercal \mathbf{T}_{lt}}{\sum_{l \in J} A_{lt}}, \mathbf{u}_{i8} = \frac{\sum_{l \in J_1} (\boldsymbol{\Omega}_{lt}^{-1}\mathbf{m}_{lt} - \frac{\mathbf{e}^\intercal \boldsymbol{\Omega}_{lt}^{-1}\mathbf{m}_{lt}}{\mathbf{e}^\intercal \boldsymbol{\Omega}_{lt}^{-1}\mathbf{e}}\boldsymbol{\Omega}_{lt}^{-1}\mathbf{e})_i \mathbf{e}^\intercal \mathbf{D}_{lt}}{\sum_{l \in J} A_{lt}},$$

$$\mathbf{u}_{i9} = \frac{\sum_{l \in J_2} A_{lt}(\boldsymbol{\Omega}_{lt}^{-1}\mathbf{m}_{lt} - \frac{\mathbf{e}^\intercal \boldsymbol{\Omega}_{lt}^{-1}\mathbf{m}_{lt}}{\mathbf{e}^\intercal \boldsymbol{\Omega}_{lt}^{-1}\mathbf{e}}\boldsymbol{\Omega}_{lt}^{-1}\mathbf{e})_i}{\sum_{l \in J} A_{lt}}.$$

We have

$$\mathbf{x}(\boldsymbol{\Lambda}_t, a, b, \lambda_I, \lambda_L)_{aggregate,i} = (1, a, b, a, b, \lambda_I, (\lambda_L - \lambda_I)a, (\lambda_L - \lambda_I)b, \lambda_L)\mathbf{u}_i(\boldsymbol{\Lambda}_t, a, b).$$

Assuming x_{it}, the share of currency i in countries' FX reserves in quarter t, satisfies

$$x_{it} = c_i + \beta \mathbf{x}(\boldsymbol{\Lambda}_t, a, b, \lambda_I, \lambda_L)_{aggregate,i} + \varepsilon_{it}, \tag{3.2}$$

where c_i is the currency-specific fixed term, capturing the time-invariant currency-specific factor in a reduced-form way. We expect c_i of euro and dollar to be much higher than that of yen and pound, reflecting the privilege of euro and dollar.[22] ε_{it} captures the unobserved shocks and the measurement errors. The solution to our portfolio choice model in Section 3.5 delivers how the time-varying component of the currency composition of FX reserves, $\beta \mathbf{x}(\boldsymbol{\Lambda}_t, a, b, \lambda_I, \lambda_L)_{aggregate,i}$, depends on the observed variables and the model's parameters. β is expected to be smaller than one, as it captures the fact that central banks are reluctant to rapidly adjust the currency composition of their FX reserves. There are several plausible reasons for that reluctance. First, there is a strong inertia bias in favor of using currency that has been the international currency in the past (Chinn and Frankel, 2005); and second, central banks prefer to see a stable composition for the reserves that would help to explain a consistent investment policy ([Ram99]).[23]

Then

$$x_{it} - \bar{x}_i = \beta(\mathbf{x}(\boldsymbol{\Lambda}_t, a, b, \lambda_I, \lambda_L)_{aggregate,i} - \bar{\mathbf{x}}(\boldsymbol{\Lambda}_t, a, b, \lambda_I, \lambda_L)_{aggregate,i}) + \varepsilon_{it} - \bar{\varepsilon}_i$$

[22] The magnitude of this term may be determined by the reserve currency issuer's relatively stable and persistent characteristics, for example, the economy's size, the size of the financial markets, and the economy's status in the world.

[23] In our baseline estimation, we implicitly assume β are the same for central banks' liquidity tranche and investment tranche. This assumption can be relaxed.

$$= \beta(1, a, b, a, b, \lambda_I, (\lambda_L - \lambda_I)a, (\lambda_L - \lambda_I)b, \lambda_L)(\mathbf{u}_i(\mathbf{\Lambda}_t, a, b) - \bar{\mathbf{u}}_i(\mathbf{\Lambda}_t, a, b)) + \varepsilon_{it} - \bar{\varepsilon}_i. \quad (3.3)$$

A GMM estimator is adopted. We use the orthogonality conditions as the moment conditions,

$$E[(\mathbf{u}_{it}(a_0, b_0) - \bar{\mathbf{u}}_i(a_0, b_0))(\varepsilon_{it} - \bar{\varepsilon}_i)] = 0,$$

$$E[\varepsilon_{it} - \bar{\varepsilon}_i] = 0,$$

which are expressed as

$$E[\mathbf{h}_i(\mathbf{x}_t, \mathbf{\Lambda}_t, a_0, b_0, \beta_0, \lambda_{I0}, \lambda_{L0})] = 0,$$

where

$$\mathbf{h}_i(\mathbf{x}_t, \mathbf{\Lambda}_t, a, b, \beta, \lambda_I, \lambda_L) = \big(\mathbf{u}_i(\mathbf{\Lambda}_t, a, b) - \bar{\mathbf{u}}_i(\mathbf{\Lambda}_t, a, b), 1\big)$$

$$\big(x_{it} - \bar{x}_i - \beta(1, a, b, a, b, \lambda_I, (\lambda_L - \lambda_I)a, (\lambda_L - \lambda_I)b, \lambda_L)(\mathbf{u}_i(\mathbf{\Lambda}_t, a, b) - \bar{\mathbf{u}}_i(\mathbf{\Lambda}_t, a, b))\big).$$

We define $\theta = (a, b, \beta, \lambda_I, \lambda_L)$, and assume the identification condition holds, i.e., θ_0 is the only solution to $E[\mathbf{h}_i(\mathbf{x}_t, \mathbf{\Lambda}_t, \theta_0)] = 0$.

A two-step estimation approach is adopted.

Step 1:

We find $\theta^* = (a^*, b^*, \beta^*, \lambda_I^*, \lambda_L^*)$ that minimizes

$$\big(\frac{1}{n}\sum_{it} \mathbf{h}_i(\mathbf{x}_t, \mathbf{\Lambda}_t, a, b, \beta, \lambda_I, \lambda_L)^\intercal\big)\mathbf{I}_n\big(\frac{1}{n}\sum_{it} \mathbf{h}_i(\mathbf{x}_t, \mathbf{\Lambda}_t, a, b, \beta, \lambda_I, \lambda_L)\big),$$

where $\mathbf{I}_n$ is the identity matrix.

We can implement this minimization in two steps.

$$x_{it} - \bar{x}_i = \beta(1, \lambda_I, \lambda_L)$$

$$\left(\begin{pmatrix} \mathbf{u}_{i1} + a(\mathbf{u}_{i2} + \mathbf{u}_{i4}) + b(\mathbf{u}_{i3} + \mathbf{u}_{i5}) \\ \mathbf{u}_{i6} - a\mathbf{u}_{i7} - b\mathbf{u}_{i8} \\ a\mathbf{u}_{i7} + b\mathbf{u}_{i8} + \mathbf{u}_{i9} \end{pmatrix} - \begin{pmatrix} \bar{\mathbf{u}}_{i1} + a(\bar{\mathbf{u}}_{i2} + \bar{\mathbf{u}}_{i4}) + b(\bar{\mathbf{u}}_{i3} + \bar{\mathbf{u}}_{i5}) \\ \bar{\mathbf{u}}_{i6} - a\bar{\mathbf{u}}_{i7} - b\bar{\mathbf{u}}_{i8} \\ a\bar{\mathbf{u}}_{i7} + b\bar{\mathbf{u}}_{i8} + \bar{\mathbf{u}}_{i9} \end{pmatrix}\right)$$

$$+\varepsilon_{it} - \bar{\varepsilon}_i.$$

The closed-form expression of (a, b), $\beta^*(a, b)$, $\lambda_I^*(a, b)$, $\lambda_L^*(a, b)$ makes the overall minimization task much more computationally feasible. The objective function above is minimized by searching for (a^*, b^*).

Step 2:

$\mathbf{W}_n$ is defined by

$$\mathbf{W}_n := \left(\frac{1}{n}\sum_{it}\mathbf{h}_i\big(\mathbf{x}_t, \mathbf{\Lambda}_t, a^*, b^*, \beta^*, \lambda_I^*, \lambda_L^*\big)\mathbf{h}_i\big(\mathbf{x}_t, \mathbf{\Lambda}_t, a^*, b^*, \beta^*, \lambda_I^*, \lambda_L^*\big)^\intercal\right)^{-1}$$

We find $\hat{\theta} = (\hat{a}, \hat{b}, \hat{\beta}, \hat{\lambda}_I, \hat{\lambda}_L)$ that minimizes

$$\big(\frac{1}{n}\sum_{it}\mathbf{h}_i(\mathbf{x}_t, \mathbf{\Lambda}_t, a, b, \lambda_I, \lambda_L)^\intercal\big)\mathbf{W}_n\big(\frac{1}{n}\sum_{it}\mathbf{h}_i(\mathbf{x}_t, \mathbf{\Lambda}_t, a, b, \lambda_I, \lambda_L)\big),$$

via the same two steps as shown in step 1, using $\mathbf{W}_n$ as the weight matrix.

The estimation results are presented in Table 3.2. More details about the estimation are shown in the Appendix 4.3.2. We can not reject the null hypothesis that $\lambda_L = 0$, which is consistent with the fact that returns are not the objective of the liquidity tranche.

Table 3.2: Estimation Results

a	b	β	λ_I	λ_L
2.6101***	2.3853***	0.5086***	0.0419***	-0.0017
(3.1603)	(2.7436)	(7.0743)	(5.9580)	(-0.5094)

t statistics in parentheses.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Figure 3.7 shows the actual and fitted euro and dollar shares in the 22 emerging and developing countries' FX reserves.[24] We can see that the fitted shares can capture well the trend and fluctuations of the actual shares.

Using the estimates, we can estimate the size of the liquidity tranche and investment tranche at the country and aggregate levels.

$$L_{aggregate\ liquidity} = \sum_{l\in J_1}\mathbf{e}^\intercal(a\mathbf{T} + b\mathbf{D}) + \sum_{l\in J_2} A_l,$$

[24] c_i is estimated by assuming the moment condition $E_t(c_i\varepsilon_{it}) = 0$ in equation (3.2), so $\hat{c}_i$ is calculated from

$$\frac{1}{T}\sum_t x_{it} = \hat{c}_i + \beta\frac{1}{T}\sum_t \mathbf{x}(\mathbf{\Lambda}_t, \hat{a}, \hat{b}, \hat{\lambda}_I, \hat{\lambda}_L)_{aggregate,i}$$

The estimates are $c_{EUR} = 0.1814, c_{JPY} = 0.0070, c_{GBP} = 0.0590, c_{USD} = 0.2439$. That indicates dollar and euro enjoy great privilege compared with yen and pound. Note the difference between c_{EUR} and c_{USD} (0.0625) is much smaller than the difference between euro's and dollar's share in the FX reserves we see in the data (between 0.3 and 0.5), indicating our model is able to explain most of the difference between euro's and dollar's share in the FX reserves.

Figure 3.7: Actual and Fitted Shares of EUR (left) and of USD (right) (2006 Q1-2014Q4)

Sources: IMF COFER; and authors' calculations.

$$L_{aggregate\ investment} = \sum_{l \in J_1} \big(A_l - \mathbf{e}^\intercal (a\mathbf{T} + b\mathbf{D})\big).$$

Compared to 2006 and 2007, there has been a significant increase in the absolute and relative sizes of the aggregate investment tranche afterwards (Figure 3.8). After 2008, despite the growth of the aggregate reserve portfolio, the absolute size of the developing countries' aggregate investment tranche has been stable, and the absolute and relative sizes of the liquidity tranche has been growing, driven by the growing imports invoicing and short-term external debt denominated in reserve currencies.

With the estimates, we can estimate the optimal currency composition of the aggregate liquidity tranche and investment tranche.

$$\mathbf{x}_{aggregate\ liquidity} = \frac{\sum_{l \in J_1} (a\mathbf{T}_l + b\mathbf{D}_l) + \sum_{l \in J_2} A_l \frac{a\mathbf{T}_l + b\mathbf{D}_l}{\mathbf{e}^\intercal (a\mathbf{T}_l + b\mathbf{D}_l)}}{\sum_{l \in J_1} \mathbf{e}^\intercal (a\mathbf{T} + b\mathbf{D}) + \sum_{l \in J_2} A_l},$$

$$\mathbf{x}_{aggregate\ investment} = \frac{\sum_{l \in J_1} \big(A_l - \mathbf{e}^\intercal (a\mathbf{T} + b\mathbf{D})\big)\big(\frac{\mathbf{\Omega}^{-1}\mathbf{e}}{\mathbf{e}^\intercal \mathbf{\Omega}^{-1}\mathbf{e}} + \lambda_I (\mathbf{\Omega}^{-1}\mathbf{m} - \frac{\mathbf{e}^\intercal \mathbf{\Omega}^{-1}\mathbf{m}}{\mathbf{e}^\intercal \mathbf{\Omega}^{-1}\mathbf{e}} \mathbf{\Omega}^{-1}\mathbf{e})\big)}{\sum_{l \in J_1} \big(A_l - \mathbf{e}^\intercal (a\mathbf{T} + b\mathbf{D})\big)}.$$

Between 2007 Q1 and 2008 Q1, the increase of the optimal euro share and the decline of the optimal dollar share in the investment tranche (Figure 3.9) are in line with the increasing returns of euro and the decreasing returns of dollar, as shown in Figure 3.5. The trend reversed after 2008.

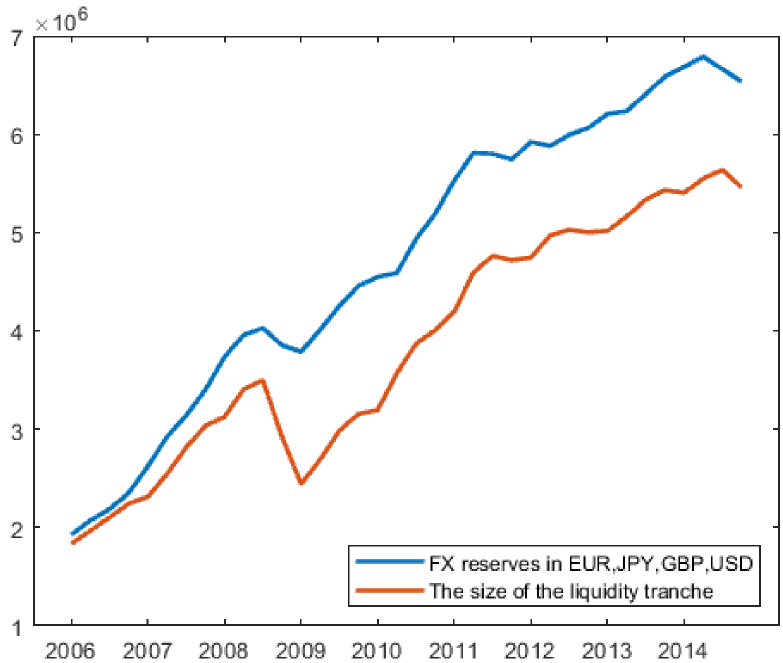

Figure 3.8: FX Reserves and the Estimated Size of the Liquidity Tranche (2006 Q1 - 2014 Q4)

Sources: Federal Reserve Bank of St. Louis; Haver; IMF COFER; and authors' calculations.

Figure 3.9: The Optimal Shares of EUR and USD in the Aggregate Liquidity Tranche and Investment Tranche (2006 - 2014)

Sources: Federal Reserve Bank of St. Louis; [Gop15]; Haver; IMF COFER and DOTS; UNCTAD; World Bank International Debt Statistics; and authors' calculations.

3.6.2 The Size of the Liquidity Tranche Included in the Reduced-form Analysis

Some hypothesis can be tested based on the relative size of the liquidity tranche,

$$\frac{\sum_{l\in J_1}\mathbf{e}^{\intercal}(a\mathbf{T}+b\mathbf{D})+\sum_{l\in J_2}A_l}{\sum_{l\in J}A_l}.$$

First, we expect that the larger the relative liquidity tranche size, the larger effect the currency compositions of imports invoicing and short-term external debt on the currency composition of FX reserves. To test the hypothesis, we interact the relative size of liquidity tranche with the currency compositions of imports invoicing and short-term external debt, and the weighted average of the currency composition in imports invoicing and short-term external debt, and include them as control variables in the regressions in Section 3.3.4. As expected, the sign of the interaction terms are positive and statistically significant (Columns (1), (3), and (4) in Table 3.3).

Second, we expect that, the larger the relative size of liquidity tranche, the smaller effect the exchange rate returns on the currency composition of FX reserves. To test it, we interact the relative size of liquidity tranche with the exchange rate returns. As expected, the sign of the interaction terms are negative and statistically significant (Columns (1)-(4) in Table 3.3).

The coefficients of the interaction terms in Table 3.3 also indicate that the estimated relative size of liquidity tranche and our estimates of the model's parameters are robust.

Table 3.3: Results of the Reduced-form Regression with the Relative Size of Liquidity Tranche Included

	(1)	(2)	(3)	(4)
imports invoicing	0.631***			
	(5.06)			
ST external debt denomination		0.383***		
		(7.23)		
imports invoicing and ST external debt denomination			0.500***	0.500***
			(8.26)	(8.24)
average return	658.0***	455.8**	551.0***	590.2***
	(3.00)	(2.57)	(3.05)	(2.83)
return volatility	-1.547	2.340	1.340	2.066
	(-0.62)	(1.15)	(0.65)	(0.73)
government bond spread				4.384
				(0.38)
imports invocing×rel liquidity size	0.0913*			
	(1.74)			
ST external debt denomination×rel liquidity size		0.0307		
		(0.67)		
imports and ST external debt×rel liquidity size			0.0726*	0.0709*
			(1.77)	(1.71)
average return×rel liquidity size	-717.1***	-416.4**	-566.4***	-614.6**
	(-2.82)	(-2.03)	(-2.71)	(-2.50)
const	0.0883***	0.151***	0.105***	0.101***
	(2.63)	(11.33)	(5.92)	(4.84)
N	144	144	144	144
within R-square	0.3024	0.5544	0.5254	0.5259

t statistics in parentheses.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

3.7 The RMB as Reserve Currency

3.7.1 The Liquidity of the Government Bond Market and the Convertibility of the Currency

When central banks decide whether to include one new currency in their FX reserves, the liquidity of that currency issuer's government bond market and the ease of convertibility of that currency are important factors. As an example, Christian Vallence's "Foreign Exchange Reserves and the Reserve Bank's Balance Sheet" describes the reserve management practice of the Reserve Bank of Australia ([Val12]),

> "The Bank first identifies eligible reserve currencies based on several criteria, including the ease of convertibility of the currency into US and/or Australian dollars, and whether the currency has underlying government bond markets that are sufficiently liquid for intervention purposes, and the sovereign issuer is of high credit quality."

We use the bid-ask spread of the government bonds to measure the liquidity of the government bond markets. Figure 3.10 shows the mean (over time) of the daily bid-ask spread of the government bond of the eurozone, Japan, UK, US, China, using the latest one-year sample up to each quarter. We can observe that China's government bonds are less liquid than other reserve currency issuers' government bonds, although the liquidity has improved since the first half of 2018 (Figure 3.10).

The bid-ask spread of each reserve currency versus domestic currency can be used to measure the transaction cost in the forex market (the ease of convertibility). Figure 3.11 shows the mean (over time) of the daily bid-ask spread of EUR, JPY, GBP, USD, and RMB (onshore market) versus the domestic currency of China, Saudi Arabia, Russia, India, Brazil, averaged for these five large emerging market economies, using the latest one-year sample up to each quarter. We can see that the transaction cost of RMB is above those of other reserve currencies in most times.

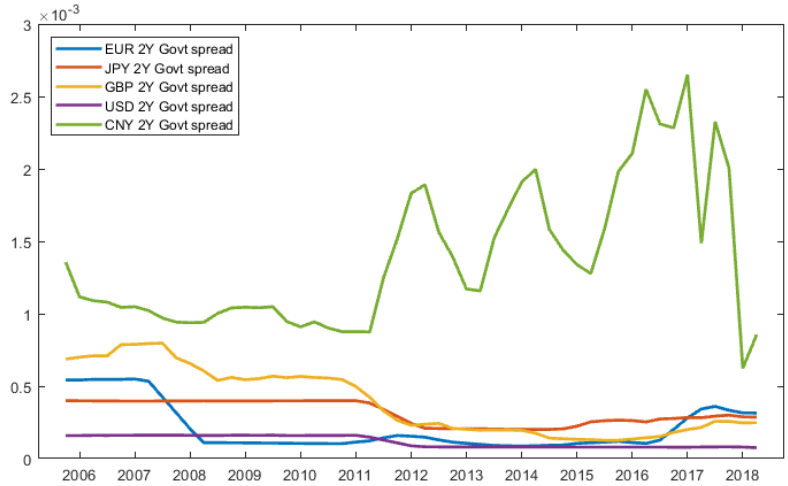

Figure 3.10: Bid-ask Spread of 2Y Government Bonds (2005 Q4 - 2018 Q2)

Sources: Bloomberg Finance L.P.; and authors' calculations.

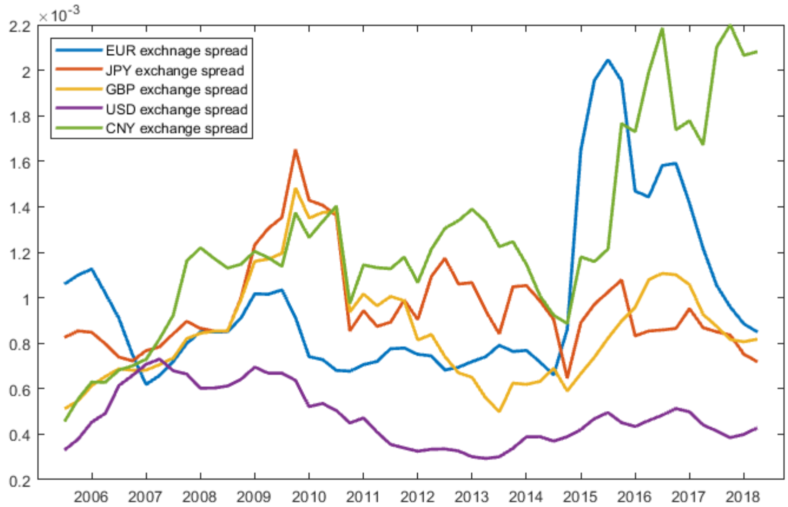

Figure 3.11: Bid-ask Spread of Exchange Rates (2005 Q3 - 2018 Q2)

Sources: Bloomberg Finance L.P.; and authors' calculations.

3.7.2 The RMB as Reserve Currency under Different Scenarios

It is expected that China will gradually enhance capital account convertibility, and the share of RMB in central banks' FX reserves will further increase from the current level of around 2 percent. Over 60 central banks or monetary authorities hold RMB reserves ([Yi18]). Based on our estimates in Section 3.6, we can estimate the share of RMB in central banks' FX reserves under different hypothetical scenarios.

The exchange rate risk and returns of RMB affects its share in the aggregate FX reserves through its share in the investment tranche.[25] As a benchmark, for each country in our sample excluding China, we set the RMB's expected return, variance and covariances with EUR, JPY, GBP, USD to be the average of SDR currencies' corresponding parameters. The currency-specific fixed term of RMB in equation (3.2) is set to be the average of SDR currencies' estimated fix terms.[26] Our analysis can be made in different scenarios, using equation (3.2) with $\mathbf{\Omega}_t, \mathbf{m}_t, \mathbf{T}_t, \mathbf{D}_t$ augmented to include RMB terms. We do the analysis for the last period in our sample, 2014 Q4.

The growing share of RMB in imports invoicing and short-term external debt

Holding other things constant, the increase in RMB's share in imports invoicing and short-term external debt will increase RMB's share in the liquidity tranche, thus in the aggregate FX reserves. This can be observed from Figure 3.12. If the share of RMB in imports invoicing and short-term external debt are 50 percent, RMB's share in FX reserves of developing countries including China would be around 15 percent;[27] RMB's share in FX reserves of developing countries excluding China would be around 35 percent.

The trend of RMB to be reference currency

For a sample comprising emerging-market economies, [SK13] find that between 2010 and 2013, RMB has increasingly become a reference currency, which they define as one that exhibits a high degree of co-movement with the domestic currencies of the reserve holders; the rise of the RMB as a reference currency is especially prominent in East Asia.[28] [MC14], and [MIC15] point out that, for China's trading partners, both changing trade invoicing and currency movements could make RMB more attractive for reserve

[25] The analysis of the exchange rate return and risk of the free floating RMB is beyond the scope of this paper.

[26] See footnote 24.

[27] We first adopt equation (3.2) to calculate the RMB's share in FX reserves of developing countries excluding China, as RMB's share in China's FX reserves is 0; then we can use the aggregate FX reserves of China and developing countries excluding China to calculate RMB's share in FX reserves of developing countries including China.

[28] As a reference currency outside Asia, the RMB has increased its presence from 7 to 11 (out of 42) countries in their sample.

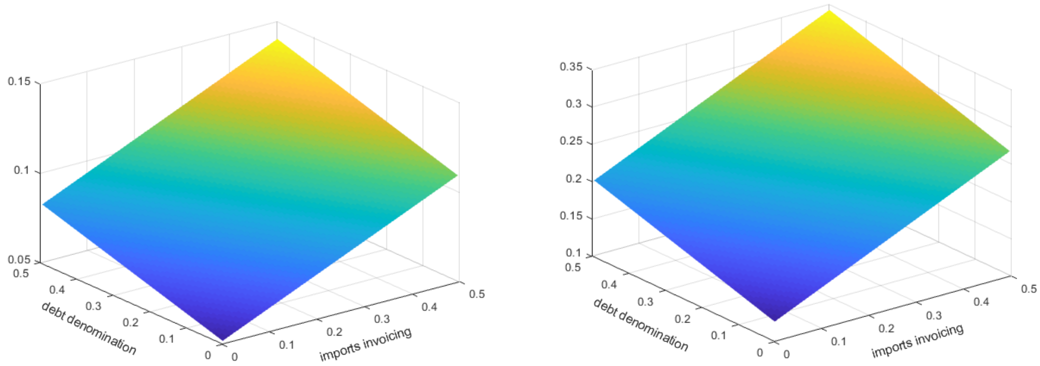

Figure 3.12: The Estimated Share of RMB in FX Reserves versus Those in Imports Invoicing (0-0.5) and Short-term External Debt (0-0.5): China Included (left) and Excluded (right)

Sources: Federal Reserve Bank of St. Louis; [Gop15]; Haver; IMF COFER and DOTs; UNCTAD; World Bank International Debt Statistics; and authors' calculations.

managers.[29]

Using our estimates, we estimate the share of RMB under the scenarios that RMB has different levels of co-movement with the domestic currencies of our sample countries, excluding China itself. If RMB has a stronger co-movement with our sample countries' domestic currencies, then from those countries' perspective, RMB is more like a risk-free asset and has a lower variance. Thus for each of our sample countries excluding China, we adjust the variance of RMB's exchange rate return we set above; the covariances between between RMB and EUR, JPY, GBP, USD are adjusted accordingly, assuming the correlation parameters are the same as before. A smaller variance of RMB implies the status of RMB as a reference currency is more prominent.

Figure 3.13 shows the estimated share of RMB in FX reserves versus the return of RMB (vary between 0.1×10^{-5} and 10×10^{-5})[30]and the average of variance of RMB

[29]The dollar zone weight (the weighted average all other economies' dollar zone weight, similar with what we did in section 3.2.3, using GDP share as weight) and share in export invoicing are highly correlated, their reduced-form analysis finds that both trade invoicing and currency movements drive changing official reserve composition, but they can not distinguish the effect of these two factors on the currency composition due to data limitation.

[30]Estimated using the sample data from 2007 Q4 to 2014 Q4, the averages of returns among our sample countries including China are around 7.18×10^{-5} for EUR, 5.44×10^{-5} for JPY, 9.26×10^{-5} for GBP, 12.54×10^{-5} for USD respectively.

among our sample countries excluding China (vary between 1.7×10^{-5} and 24×10^{-5})[31]. We can see a more prominent status as reference currency could lead to higher RMB's share in FX reserves; the magnitude of the effect is more significant when RMB has a relatively higher return.

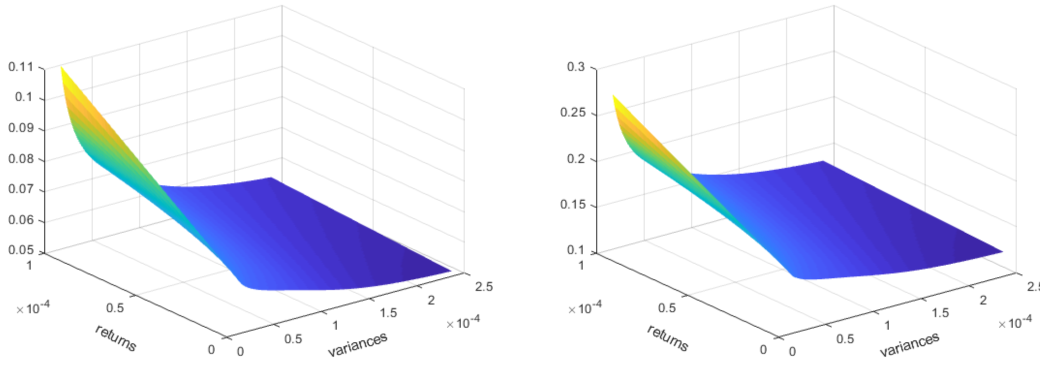

Figure 3.13: The Estimated Share of RMB in FX Reserves versus the Returns of RMB (0.1×10^{-5}-10×10^{-5}) and the Variances of RMB (1.7×10^{-5}-24×10^{-5}): China Included (left) and Excluded (right)

Sources: Federal Reserve Bank of St. Louis; [Gop15]; Haver; IMF COFER and DOTS; UNCTAD; World Bank International Debt Statistics; and authors' calculations.

3.8 Conclusion

The way central banks manage their reserves have evolved over the past decades, reflecting changes in both the economic and the broader institutional environment. Central banks have begun to manage their FX reserves in two or more tranches, to satisfy their FX liquidity needs in the liquidity tranche, and to pursue higher returns subject to some risk constraints in the investment tranche. The appearance of the investment tranche reflects the continuing accumulation of reserves and central banks' growing emphasis on preserving and enhancing the value of the reserves.

In this paper, based on the results of a reduced-form analysis and country-level cross-

[31] Estimated using the sample data from 2007 Q4 to 2014 Q4, the averages of variances among our sample countries including China are around 5.23×10^{-5} for EUR, 40.71×10^{-5} for JPY, 5.98×10^{-5} for GBP, 4.97×10^{-5} for USD respectively.

sectional evidence, we present the importance of the currency compositions of imports invoicing and external debt, and exchange rate risk and returns for the currency composition of FX reserves.

Based on central banks' practice and literature review, we have developed a central bank's reserve portfolio choice model to analyze the determinants of the currency composition of FX reserves. Our model formalizes central banks' reserve tranching practice and is estimated to quantify the roles of the currency compositions of imports invoicing and short-term external debt, and risk and returns of exchange rates in determining the currency composition of FX reserves at the aggregate level of central banks. That is a key contribution of our paper, given the absence of structural models in the literature.

Based on our estimates, we find that compared to 2006 and 2007, there had been a significant increase of the absolute and relative sizes of the investment tranche in emerging and developing countries' FX reserve. After 2008, despite the growth of the aggregate reserve portfolio, the absolute size of the estimated investment tranche has been stable, likely due to the growing liquidity needs associated with the growing imports invoicing and short-term external debt denominated in reserve currencies.

As expected, our estimates show that the investment tranche does have returns objective, while the liquidity tranche does not. The larger the relative size of the liquidity tranche, the more important the effect of the imports invoicing and short-term external debt, and the smaller the effect of reserve currencies' returns on the currency composition of FX reserves. The choice of numeraire is relevant in particular for the investment tranche, while the currency composition of liquidity tranche is determined by the currency composition of FX liquidity needs. There is tendency for central banks to use domestic currency as numeraire to measure the worth of reserve assets, which is the assumption we have used in the estimation.

Finally, we apply our model to estimate the likely paths of share of RMB in FX reserves under different scenarios. The scenarios include a larger share of RMB in countries' imports invoicing and short-term external debt, and the trend of RMB becoming reference currency for other countries. The exercise helps to shed light on the potential status of the RMB as an international currency.

CHAPTER 4

Appendix

4.1 Appendix of Chapter 1

4.1.1 Proofs of Propositions and Lemmas

Proof of Proposition 1

Proof.

Dealer 1's trading strategy is $Q^1_{1,2} = a^1_{1,2}s_1 + b^1_{1,2}p + c^1_{1,2}\eta_1$, dealer 2's trading strategy is $Q^2_{1,2} = b^2_{1,2}p + c^2_{1,2}\eta_2$. For dealer 1, market clearing implies

$$Q^1_{1,2} + b^2_{1,2}p + c^2_{1,2}\eta_2 + \beta p = 0,$$

thus

$$p = -\frac{c^2_{1,2}\eta_2}{b^2_{1,2} + \beta} - \frac{Q^1_{1,2}}{b^2_{1,2} + \beta} := I_1 + \lambda^1_{1,2}Q^1_{1,2}.$$

Dealer 1's optimization problem is

$$\max_{Q^1_{1,2}}(\mathbb{E}(\theta^1_{1,2}|s_1, \eta_1) - p)Q^1_{1,2} = (\mathbb{E}(\theta^1_{1,2}|s_1, \eta_1) - I_1 - \lambda^1_{1,2}Q^1_{1,2})Q^1_{1,2}.$$

FOC

$$-2\lambda^1_{1,2}Q^1_{1,2} + \mathbb{E}(\theta^1_{1,2}|s_1, \eta_1) - I_1$$

$$= -2\lambda^1_{1,2}Q^1_{1,2} + \mathbb{E}(\theta^1_{1,2}|s_1, \eta_1) - p + \lambda^1_{1,2}Q^1_{1,2} = -\lambda^1_{1,2}Q^1_{1,2} + \mathbb{E}(\theta^1_{1,2}|s_1, \eta_1) - p = 0,$$

thus

$$Q^1_{1,2} = \frac{\mathbb{E}(\theta^1_{1,2}|s_1, \eta_1) - p}{\lambda^1_{1,2}},$$

$$\begin{pmatrix} \theta \\ s_1 \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_\theta & \sigma^2_\theta \\ \sigma^2_\theta & \sigma^2_\theta + \sigma^2_\varepsilon \end{pmatrix}\right],$$

thus

$$\mathbb{E}(\theta|s_1) = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2} s_1,$$

$$\mathbb{E}(\theta_{1,2}^1|s_1, \eta_1) = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2} s_1 + \eta_1,$$

we have

$$Q_{1,2}^1 = -(b_{1,2}^2 + \beta)(\frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2} s_1 + \eta_1 - p). \tag{4.1.1}$$

For dealer 2, market clearing condition implies

$$Q_{1,2}^2 + a_{1,2}^1 s_1 + b_{1,2}^1 p + c_{1,2}^1 \eta_1 + \beta p = 0,$$

thus

$$p = -\frac{a_{1,2}^1 s_1 + c_{1,2}^1 \eta_1}{b_{1,2}^1 + \beta} - \frac{Q_{1,2}^2}{b_{1,2}^1 + \beta} := I_2 + \lambda_{1,2}^2 Q_{1,2}^2.$$

Dealer 2's optimization problem is

$$\max_{Q_{1,2}^2} (\mathbb{E}(\theta_2|I_2, \eta_2) - p) Q_{1,2}^2 = (\mathbb{E}(\theta_2|I_2, \eta_2) - I_2 - \lambda_{1,2}^2 Q_{1,2}^2) Q_{1,2}^2.$$

FOC implies

$$-2\lambda_{1,2}^2 Q_{1,2}^2 + \mathbb{E}(\theta_2|I_2, \eta_2) - I_2 = 0,$$

$$\begin{pmatrix} \theta \\ I_2(-\frac{b_{1,2}^1+\beta}{a_{1,2}^1}) \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 \\ \sigma_\theta^2 & \sigma_\theta^2 + \sigma_\varepsilon^2 + \frac{(c_{1,2}^1)^2}{(a_{1,2}^1)^2}\sigma_\eta^2 \end{pmatrix}\right],$$

thus

$$\mathbb{E}(\theta|I_2) = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2 + \frac{(c_{1,2}^1)^2}{(a_{1,2}^1)^2}\sigma_\eta^2} I_2(-\frac{b_{1,2}^1 + \beta}{a_{1,2}^1}),$$

$$\mathbb{E}(\theta_2|I_2, \eta_2) = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2 + \frac{(c_{1,2}^1)^2}{(a_{1,2}^1)^2}\sigma_\eta^2} I_2(-\frac{b_{1,2}^1 + \beta}{a_{1,2}^1}) + \eta_2,$$

thus

$$-2\lambda_{1,2}^2 Q_{1,2}^2 + \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2 + \frac{(c_{1,2}^1)^2}{(a_{1,2}^1)^2}\sigma_\eta^2} I_2(-\frac{b_{1,2}^1 + \beta}{a_{1,2}^1}) + \eta_2 - I_2 = 0,$$

thus

$$-2\lambda_{1,2}^2 Q_{1,2}^2 + (\frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2 + \frac{(c_{1,2}^1)^2}{(a_{1,2}^1)^2}\sigma_\eta^2} (-\frac{b_{1,2}^1 + \beta}{a_{1,2}^1}) - 1)(p - \lambda_{1,2}^2 Q_{1,2}^2) + \eta_2 = 0,$$

thus

$$\lambda_{1,2}^2 Q_{1,2}^2\left(\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2+\frac{(c_{1,2}^1)^2}{(a_{1,2}^1)^2}\sigma_\eta^2}\left(-\frac{b_{1,2}^1+\beta}{a_{1,2}^1}\right)+1\right)=\left(\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2+\frac{(c_{1,2}^1)^2}{(a_{1,2}^1)^2}\sigma_\eta^2}\left(-\frac{b_{1,2}^1+\beta}{a_{1,2}^1}\right)-1\right)p+\eta_2,$$

we have

$$Q_{1,2}^2=-(b_{1,2}^1+\beta)\left(\frac{\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2+\frac{(c_{1,2}^1)^2}{(a_{1,2}^1)^2}\sigma_\eta^2}\left(-\frac{b_{1,2}^1+\beta}{a_{1,2}^1}\right)-1}{\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2+\frac{(c_{1,2}^1)^2}{(a_{1,2}^1)^2}\sigma_\eta^2}\left(-\frac{b_{1,2}^1+\beta}{a_{1,2}^1}\right)+1}p+\frac{1}{\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2+\frac{(c_{1,2}^1)^2}{(a_{1,2}^1)^2}\sigma_\eta^2}\left(-\frac{b_{1,2}^1+\beta}{a_{1,2}^1}\right)+1}\eta_2\right). \quad (4.1.2)$$

From (4.1.1)(4.1.2), we have

$$a_{1,2}^1=-(b_{1,2}^2+\beta)\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}:=-(b_{1,2}^2+\beta)\frac{1}{1+\gamma}$$

$$b_{1,2}^1=b_{1,2}^2+\beta,$$

$$c_{1,2}^1=-(b_{1,2}^2+\beta),$$

$$b_{1,2}^2=-(b_{1,2}^1+\beta)\frac{\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2+\frac{(c_{1,2}^1)^2}{(a_{1,2}^1)^2}\sigma_\eta^2}\left(-\frac{b_{1,2}^1+\beta}{a_{1,2}^1}\right)-1}{\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2+\frac{(c_{1,2}^1)^2}{(a_{1,2}^1)^2}\sigma_\eta^2}\left(-\frac{b_{1,2}^1+\beta}{a_{1,2}^1}\right)+1},$$

$$c_{1,2}^2=-(b_{1,2}^1+\beta)\frac{1}{\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2+\frac{(c_{1,2}^1)^2}{(a_{1,2}^1)^2}\sigma_\eta^2}\left(-\frac{b_{1,2}^1+\beta}{a_{1,2}^1}\right)+1},$$

then we have

$$b_{1,2}^2\left(\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2+\frac{(c_{1,2}^1)^2}{(a_{1,2}^1)^2}\sigma_\eta^2}\left(-\frac{b_{1,2}^1+\beta}{a_{1,2}^1}\right)+1\right)=-(b_{1,2}^1+\beta)\left(\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2+\frac{(c_{1,2}^1)^2}{(a_{1,2}^1)^2}\sigma_\eta^2}\left(-\frac{b_{1,2}^1+\beta}{a_{1,2}^1}\right)-1\right),$$

thus

$$b_{1,2}^2\left(\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2+\frac{\sigma_\eta^2}{(\frac{1}{1+\gamma})^2}}\left(-\frac{b_{1,2}^2+2\beta}{-(b_{1,2}^2+\beta)\frac{1}{1+\gamma}}\right)+1\right)$$

$$=-(b_{1,2}^2+2\beta)\left(\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2+\frac{\sigma_\eta^2}{(\frac{1}{1+\gamma})^2}}\left(-\frac{b_{1,2}^2+2\beta}{-(b_{1,2}^2+\beta)\frac{1}{1+\gamma}}\right)-1\right).$$

Define $\kappa_1\equiv\frac{\sigma_\eta^2}{Var(\theta)-Var(\theta|s_1)}=\varphi(1+\gamma)$, we have

$$b_{1,2}^2\left(\frac{1}{1+\kappa_1}\frac{b_{1,2}^2+2\beta}{b_{1,2}^2+\beta}+1\right)=-(b_{1,2}^2+2\beta)\left(\frac{1}{1+\kappa_1}\frac{b_{1,2}^2+2\beta}{b_{1,2}^2+\beta}-1\right), \quad (4.1.3)$$

thus

$$b^2_{1,2}(\frac{1}{1+\kappa_1}(b^2_{1,2}+2\beta)+b^2_{1,2}+\beta)=-(b^2_{1,2}+2\beta)(\frac{1}{1+\kappa_1}(b^2_{1,2}+2\beta)-(b^2_{1,2}+\beta)),$$

thus

$$\frac{1}{1+\kappa_1}(b^2_{1,2}+2\beta)b^2_{1,2}+(b^2_{1,2}+\beta)b^2_{1,2}=-(b^2_{1,2}+2\beta)^2\frac{1}{1+\kappa_1}+(b^2_{1,2}+2\beta)(b^2_{1,2}+\beta),$$

thus

$$\frac{1}{1+\kappa_1}(b^2_{1,2}+2\beta)2(b^2_{1,2}+\beta)=(b^2_{1,2}+\beta)2\beta,$$

we have

$$b^2_{1,2}=\beta(1+\kappa_1)-2\beta=\beta(\kappa_1-1)=\beta(\varphi(1+\gamma)-1).$$

We require $\lambda^1_{1,2}=-\frac{1}{b^2_{1,2}+\beta}>0$, $\lambda^2_{1,2}=-\frac{1}{b^1_{1,2}+\beta}>0$. We immediately have $b^1_{1,2}+\beta=b^2_{1,2}+2\beta=\beta(1+\kappa_1)<0$, $b^2_{1,2}+\beta<0$.

Then we have

$$a^1_{1,2}=-\kappa_1\beta\frac{1}{1+\gamma}=-\beta\varphi,$$

$$b^1_{1,2}=\kappa_1\beta=\beta\varphi(1+\gamma),$$

$$c^1_{1,2}=-\kappa_1\beta=-\beta\varphi(1+\gamma),$$

$$c^2_{1,2}=b^2_{1,2}\frac{1}{\frac{\sigma^2_\theta}{\sigma^2_\theta+\sigma^2_\varepsilon+\frac{(c^1_{1,2})^2}{(a^1_{1,2})^2}\sigma^2_\eta}(-\frac{b^1_{1,2}+\beta}{a^1_{1,2}})-1}=(\beta(1+\kappa_1)-2\beta)\frac{1}{\frac{\sigma^2_\theta}{\sigma^2_\theta+\sigma^2_\varepsilon+\frac{\sigma^2_\eta}{(\frac{1}{1+\gamma})^2}}(-\frac{\beta(1+\kappa_1)}{-(\beta(1+\kappa_1)-\beta)\frac{1}{1+\gamma}})-1}$$

$$=(\beta(1+\kappa_1)-2\beta)\frac{1}{\frac{1}{1+\kappa_1}\frac{1}{1+\gamma}(-\frac{\beta(1+\kappa_1)}{-(\beta(1+\kappa_1)-\beta)\frac{1}{1+\gamma}})-1}=-\kappa_1\beta=-\beta\varphi(1+\gamma).$$

Using $a^1_{1,2}s_1+b^1_{1,2}p+c^1_{1,2}\eta_1+b^2_{1,2}p+c^2_{1,2}\eta_2+\beta p=0$, we have

$$p=\frac{\varphi}{2\kappa_1}s_1+\frac{\eta_1+\eta_2}{2}.$$

■

Proof of Lemma 1

Proof.

Following the same procedure in the proof of Proposition 1, we have the counterparts of (4.1.1)(4.1.2) above,

$$Q^i_{i,i+1}=-(b^{i+1}_{i,i+1}+\beta)(\frac{\sigma^2_\theta}{Var(s_i)}s_i+\eta_i-p),$$

$$Q_{i,i+1}^{i+1} = -(b_{i,i+1}^{i}+\beta)(\frac{\frac{\sigma_\theta^2}{Var(s_i)+\frac{(c_{i,i+1}^i)^2}{(a_{i,i+1}^i)^2}\sigma_\eta^2}(-\frac{b_{i,i+1}^i+\beta}{a_{i,i+1}^i})-1}{\frac{\sigma_\theta^2}{Var(s_i)+\frac{(c_{i,i+1}^i)^2}{(a_{i,i+1}^i)^2}\sigma_\eta^2}(-\frac{b_{i,i+1}^i+\beta}{a_{i,i+1}^i})+1}p+\frac{1}{\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2+\frac{(c_{i,i+1}^i)^2}{(a_{i,i+1}^i)^2}\sigma_\eta^{i+1}}(-\frac{b_{i,i+1}^i+\beta}{a_{i,i+1}^i})+1}\eta_{i+1}).$$

we have

$$b_{i,i+1}^i = b_{i,i+1}^{i+1}+\beta,$$

$$b_{i,i+1}^{i+1} = -(b_{i,i+1}^i+\beta)(\frac{\frac{\sigma_\theta^2}{Var(s_i)+\frac{(c_{i,i+1}^i)^2}{(a_{i,i+1}^i)^2}\sigma_\eta^2}(-\frac{b_{i,i+1}^i+\beta}{a_{i,i+1}^i})-1}{\frac{\sigma_\theta^2}{Var(s_i)+\frac{(c_{i,i+1}^i)^2}{(a_{i,i+1}^i)^2}\sigma_\eta^2}(-\frac{b_{i,i+1}^i+\beta}{a_{i,i+1}^i})+1}),$$

thus

$$b_{i,i+1}^{i+1}(\frac{\sigma_\theta^2}{Var(s_i)+\frac{(c_{i,i+1}^i)^2}{(a_{i,i+1}^i)^2}\sigma_\eta^2}(-\frac{b_{i,i+1}^i+\beta}{a_{i,i+1}^i})+1)$$

$$= -(b_{i,i+1}^{i+1}+2\beta)(\frac{\sigma_\theta^2}{Var(s_i)+\frac{(c_{i,i+1}^i)^2}{(a_{i,i+1}^i)^2}\sigma_\eta^2}(-\frac{b_{i,i+1}^i+\beta}{a_{i,i+1}^i})-1),$$

thus

$$b_{i,i+1}^{i+1}(\frac{\sigma_\theta^2}{Var(s_i)+\frac{(Var(s_i))^2}{(\sigma_\theta^2)^2}\sigma_\eta^2}(-\frac{b_{i,i+1}^i+\beta}{-(b_{i,i+1}^{i+1}+\beta)\frac{\sigma_\theta^2}{Var(s_i)}})+1)$$

$$= -(b_{i,i+1}^{i+1}+2\beta)(\frac{\sigma_\theta^2}{Var(s_i)+\frac{(Var(s_i))^2}{(\sigma_\theta^2)^2}\sigma_\eta^2}(-\frac{b_{i,i+1}^i+\beta}{-(b_{i,i+1}^{i+1}+\beta)\frac{\sigma_\theta^2}{Var(s_i)}})-1).$$

Using

$$\frac{1}{1+\kappa_i} = \frac{1}{1+\frac{\sigma_\eta^2}{Var(\theta)-Var(\theta|s_i)}} = \frac{1}{1+\frac{\sigma_\eta^2}{\frac{\sigma_\theta^4}{Var(s_i)}}},$$

we have the counterpart of (4.1.3) above

$$b_{i,i+1}^{i+1}(\frac{1}{1+\kappa_i}(\frac{b_{i,i+1}^{i+1}+2\beta}{(b_{i,i+1}^{i+1}+\beta)})+1) = -(b_{i,i+1}^{i+1}+2\beta)(\frac{1}{1+\kappa_i}(\frac{b_{i,i+1}^{i+1}+2\beta}{(b_{i,i+1}^{i+1}+\beta)})-1).$$

The rest of the proof is the same as the proof of Proposition 1.

■

Proof of Lemma 2

Proof.

For dealer 2, market clearing condition for link (1,2) implies

$$Q_{1,2}^2 + a_{1,2}^1 s_1 + b_{1,2}^1 p_{1,2} + c_{1,2}^1 \eta_1 + \beta p_{1,2} = 0,$$

thus

$$p_{1,2} = -\frac{a_{1,2}^1 s_1 + c_{1,2}^1 \eta_1}{b_{1,2}^1 + \beta} - \frac{Q_{1,2}^2}{b_{1,2}^1 + \beta} := I_2 + \lambda_{1,2}^2 Q_{1,2}^2.$$

For dealer 2, $p_{1,2}$ is informationally equivalent to $s_1 + \frac{c_{1,2}^1}{a_{1,2}^1}\eta_1 = s_1 + \frac{\kappa_1}{\varphi}\eta_1 := s_2$. We have

$$Var(\theta|s_2) = \sigma_\theta^2 - \frac{\sigma_\theta^4}{Var(s_1) + (\frac{\kappa_1}{\varphi})^2\sigma_\eta^2} = \sigma_\theta^2 - \frac{\sigma_\theta^4}{\frac{\kappa_1}{\varphi^2}\sigma_\eta^2 + (\frac{\kappa_1}{\varphi})^2\sigma_\eta^2},$$

thus

$$\frac{\sigma_\eta^2}{Var(\theta) - Var(\theta|s_2)} = \frac{\sigma_\eta^2}{\frac{\sigma_\theta^4}{\frac{\kappa_1}{\varphi^2}\sigma_\eta^2 + (\frac{\kappa_1}{\varphi})^2\sigma_\eta^2}} = \kappa_1 + \kappa_1^2,$$

$$\kappa_2 \equiv \frac{\sigma_\eta^2}{Var(\theta) - Var(\theta|s_2)} = \kappa_1 + \kappa_1^2.$$

Using

$$\kappa_2 = \kappa_1 + \kappa_1^2,$$

$$\frac{c_{2,3}^2}{a_{2,3}^2} = \frac{\kappa_2}{\varphi},$$

we prove by induction. Suppose for any $2 \leq j < i$,

$$\kappa_j = \kappa_{j-1} + \kappa_{j-1}^2,$$

$$\frac{c_{j,j+1}^j}{a_{j,j+1}^j} = \frac{\kappa_j}{\varphi},$$

thus for dealer i, $p_{i-1,i}$ is informationally equivalent to $s_{i-1} + \frac{c_{i-1,i}^{i-1}}{a_{i-1,i}^{i-1}}\eta_{i-1} = s_{i-1} + \frac{\kappa_{i-1}}{\varphi}\eta_{i-1} := s_i$, thus

$$Var(\theta|s_i) = \sigma_\theta^2 - \frac{\sigma_\theta^4}{Var(s_i)} = \sigma_\theta^2 - \frac{\sigma_\theta^4}{Var(s_{i-1}) + (\frac{\kappa_{i-1}}{\varphi})^2\sigma_\eta^2},$$

then we have

$$\kappa_i = \frac{\sigma_\eta^2}{Var(\theta) - Var(\theta|s_i)} = \frac{\sigma_\eta^2}{\frac{\sigma_\theta^4}{Var(s_{i-1}) + (\frac{\kappa_{i-1}}{\varphi})^2\sigma_\eta^2}} = (Var(s_{i-1}) + (\frac{\kappa_{i-1}}{\varphi})^2\sigma_\eta^2)\frac{\varphi}{\sigma_\theta^2}$$

$$= (Var(s_{i-2}) + (\frac{\kappa_{i-2}}{\varphi})^2\sigma_\eta^2 + (\frac{\kappa_{i-1}}{\varphi})^2\sigma_\eta^2)\frac{\varphi}{\sigma_\theta^2} = (Var(s_1) + (\frac{\kappa_1}{\varphi})^2\sigma_\eta^2 ... + (\frac{\kappa_{i-2}}{\varphi})^2\sigma_\eta^2 + (\frac{\kappa_{i-1}}{\varphi})^2\sigma_\eta^2)\frac{\varphi}{\sigma_\theta^2}$$

$$= (\frac{\kappa_1}{\varphi^2}\sigma_\eta^2 + (\frac{\kappa_1}{\varphi})^2\sigma_\eta^2 ... + (\frac{\kappa_{i-2}}{\varphi})^2\sigma_\eta^2 + (\frac{\kappa_{i-1}}{\varphi})^2\sigma_\eta^2)\frac{\varphi}{\sigma_\theta^2} = \kappa_{i-1} + \kappa_{i-1}^2.$$

■

Proof of Lemma 3

Proof.

Dealer's trading profit is

$$\mathbb{E}[(Q_{i,i+1}^{i+1}(\eta_{i+1}-\eta_i)] + \mathbb{E}[\beta p_{i,i+1}(p_{i,i+1}-\theta-\eta_i)],$$

where

$$Q_{i,i+1}^{i+1} = b_{i,i+1}^{i+1} p_{i,i+1} + c_{i,i+1}^{i+1}\eta_{i+1},$$
$$p_{i,i+1} = \frac{\varphi}{2\kappa_i}s_i + \frac{\eta_i+\eta_{i+1}}{2},$$
$$c_{i,i+1}^{i} = c_{i,i+1}^{i+1} = -\beta\kappa_i.$$

We have

$$\mathbb{E}[p_{i,i+1}(\eta_{i+1}-\eta_i)] = 0,$$
$$\mathbb{E}[p_{i,i+1}^2] = \frac{\varphi^2}{4\kappa_i^2}Var(s_i) + \frac{\sigma_\eta^2}{2} = \frac{\varphi^2}{4\kappa_i^2}\frac{\kappa_i}{\varphi^2}\sigma_\eta^2 + \frac{\sigma_\eta^2}{2} = \frac{\sigma_\eta^2}{4\kappa_i} + \frac{\sigma_\eta^2}{2},$$

thus

$$\mathbb{E}[(Q_{i,i+1}^{i+1}(\eta_{i+1}-\eta_i)] = c_{i,i+1}^{i+1}\sigma_\eta^2 = -\beta\sigma_\eta^2\kappa_i,$$
$$\mathbb{E}[\beta p_{i,i+1}(p_{i,i+1}-\theta-\eta_i)] = \beta(\frac{\sigma_\eta^2}{4\kappa_i} + \frac{\sigma_\eta^2}{2} - \frac{\varphi}{2\kappa_i}\sigma_\theta^2 - \frac{\sigma_\eta^2}{2}) = -\beta\frac{\sigma_\eta^2}{4\kappa_i}.$$

■

Proof of Lemma 4

Proof.

$$p_{i,i+1} = \varphi\frac{1}{2\kappa_i}s_i + \frac{\eta_i+\eta_{i+1}}{2},$$

thus

$$p_{i,i+1} - p_{i-1,i} = \varphi\frac{1}{2\kappa_i}s_i + \frac{\eta_i+\eta_{i+1}}{2} - (\varphi\frac{1}{2\kappa_{i-1}}s_{i-1} + \frac{\eta_{i-1}+\eta_n}{2})$$
$$= \varphi\frac{1}{2\kappa_i}(s_1 + \kappa_1\frac{\sigma_\theta^2}{\sigma_\eta^2}\eta_1 + \kappa_2\frac{\sigma_\theta^2}{\sigma_\eta^2}\eta_2 + ... + \kappa_{i-1}\frac{\sigma_\theta^2}{\sigma_\eta^2}\eta_{i-1})$$
$$-\varphi\frac{1}{2\kappa_{i-1}}(s_1 + \kappa_1\frac{\sigma_\theta^2}{\sigma_\eta^2}\eta_1 + \kappa_2\frac{\sigma_\theta^2}{\sigma_\eta^2}\eta_2 + ... + \kappa_{n-2}\frac{\sigma_\theta^2}{\sigma_\eta^2}\eta_{n-2}) + \frac{\eta_{i+1}-\eta_{i-1}}{2}$$
$$= \varphi\frac{1}{2}(\frac{1}{\kappa_i} - \frac{1}{\kappa_{i-1}})(s_1 + \kappa_1\frac{\sigma_\theta^2}{\sigma_\eta^2}\eta_1 + ...\kappa_{n-2}\frac{\sigma_\theta^2}{\sigma_\eta^2}\eta_{n-2}) + \frac{1}{2}\frac{\kappa_{i-1}}{\kappa_i}\eta_{i-1} + \frac{\eta_{i+1}-\eta_{i-1}}{2}$$
$$= \varphi\frac{1}{2}(-\frac{1}{\kappa_{i-1}+1})s_1 + \frac{1}{2}(-\frac{1}{\kappa_{i-1}+1})(\kappa_1\eta_1 + ...\kappa_{n-2}\eta_{n-2}) + (\frac{1}{2}\frac{1}{\kappa_{i-1}+1} - \frac{1}{2})\eta_{i-1} + \frac{1}{2}\eta_{i+1},$$

thus

$$Var(p_{i,i+1}-p_{i-1,i}) = (\varphi\frac{1}{2})^2\frac{\sigma_\theta^2(1+\gamma)}{(\kappa_{i-1}+1)^2}+(\frac{1}{2}\frac{1}{\kappa_{i-1}+1})^2(\kappa_1^2+..+\kappa_{n-2}^2)\sigma_\eta^2+(\frac{1}{2}\frac{\kappa_{i-1}}{\kappa_{i-1}+1})^2\sigma_\eta^2+\frac{1}{4}\sigma_\eta^2$$

$$= (\varphi\frac{1}{2})^2\frac{\sigma_\theta^2\kappa_1\frac{\sigma_\theta^2}{\sigma_\eta^2}}{(\kappa_{i-1}+1)^2} + (\frac{1}{2}\frac{1}{\kappa_{i-1}+1})^2(\kappa_1^2+..+\kappa_{n-2}^2)\sigma_\eta^2 + (\frac{1}{2}\frac{\kappa_{i-1}}{\kappa_{i-1}+1})^2\sigma_\eta^2 + \frac{1}{4}\sigma_\eta^2$$

$$= \frac{\sigma_\eta^2}{4}\frac{1}{(\kappa_{i-1}+1)^2}(\kappa_1+\kappa_1^2+..+\kappa_{n-2}^2) + (\frac{1}{2}\frac{\kappa_{i-1}}{\kappa_{i-1}+1})^2\sigma_\eta^2 + \frac{1}{4}\sigma_\eta^2$$

$$= \frac{\sigma_\eta^2}{4}\frac{1}{(\kappa_{i-1}+1)^2}\kappa_{i-1}+(\frac{1}{2}\frac{\kappa_{i-1}}{\kappa_{i-1}+1})^2\sigma_\eta^2+\frac{1}{4}\sigma_\eta^2 = \frac{\sigma_\eta^2}{4}\frac{1}{(\kappa_{i-1}+1)^2}\kappa_i+\frac{1}{4}\sigma_\eta^2 = \frac{\sigma_\eta^2}{4}\frac{\kappa_{i-1}}{\kappa_{i-1}+1}+\frac{1}{4}\sigma_\eta^2,$$

thus given any κ_1, $Var(p_{i,i+1}-p_{i-1,i})$ is increasing in $i \geq 2$. From $\mathbb{E}(|x|) = \sigma\sqrt{\frac{2}{\pi}}\exp(-\frac{\mu^2}{2\sigma^2})+\mu\big(1-2\Phi(-\frac{\mu}{\sigma})\big)$,

$$\frac{d\mathbb{E}(|x|)}{d\sigma} = \sqrt{\frac{2}{\pi}}\exp(-\frac{\mu^2}{2\sigma^2}) > 0,$$

we have $\mathbb{E}(|p_{i,i+1}-p_{i-1,i}|)$ is increasing in $i \geq 2$.

■

Proof of Proposition 2

Proof.

Dealer i's trading strategy is $Q_{i,i+1}^i = a_{i,i+1}^i\hat{s}_i + b_{i,i+1}^i p + c_{i,i+1}^i\eta_i +_{i,i+1}^i S$, dealer $i+1$'s trading strategy is $Q_{i,i+1}^{i+1} = b_{i,i+1}^{i+1}p + c_{i,i+1}^{i+1}\eta_{i+1} + d_{i,i+1}^{i+1}S$. For dealer i, market clearing implies

$$Q_{i,i+1}^i + b_{i,i+1}^{i+1}p + c_{i,i+1}^{i+1}\eta_{i+1} + d_{i,i+1}^{i+1}S + \beta p = 0,$$

thus

$$p = -\frac{c_{i,i+1}^{i+1}\eta_{i+1} + d_{i,i+1}^{i+1}S}{b_{i,i+1}^{i+1}+\beta} - \frac{Q_{i,i+1}^i}{b_{i,i+1}^{i+1}+\beta} := I_i + \lambda_{i,i+1}^i Q_{i,i+1}^i.$$

Dealer i's optimization problem is

$$\max_{Q_{i,i+1}^i}(\mathbb{E}(\theta_i|\hat{s}_i,\eta_i,S)-p)Q_{i,i+1}^i = (\mathbb{E}(\theta_i|s_i,\eta_i,S)-I_i-\lambda_{i,i+1}^i Q_{i,i+1}^i)Q_{i,i+1}^i,$$

FOC

$$-2\lambda_{i,i+1}^i Q_{i,i+1}^i + \mathbb{E}(\theta_i|s_i,\eta_i,S) - I_i = -2\lambda_{i,i+1}^i Q_{i,i+1}^i + \mathbb{E}(\theta_i|s_i,\eta_i,S) - p + \lambda_{i,i+1}^i Q_{i,i+1}^i$$

$$= -\lambda_{i,i+1}^i Q_{i,i+1}^i + \mathbb{E}(\theta_i|\hat{s}_i,\eta_i,S) - p = 0,$$

thus

$$Q^i_{i,i+1} = \frac{\mathbb{E}(\theta_i|s_i, \eta_i, S) - p}{\lambda^i_{i,i+1}}.$$

Using projection theorem,

$$\begin{pmatrix} \theta \\ \hat{s}_i \\ S \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_\theta & \sigma^2_\theta & \sigma^2_\theta \\ \sigma^2_\theta & Var(\hat{s}_i) & Cov(\hat{s}_i, S) \\ \sigma^2_\theta & Cov(\hat{s}_i, S) & Var(S) \end{pmatrix}\right],$$

$$\mathbb{E}(\theta_i|\hat{s}_i, S, \eta_i) = \frac{\sigma^2_\theta(Var(S) - Cov(\hat{s}_i, S))}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}\hat{s}_i + \frac{\sigma^2_\theta(Var(\hat{s}_i) - Cov(\hat{s}_i, S))}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}S + \eta_i,$$

we have

$$Q^i_{i,i+1} = -(b^{i+1}_{i,i+1} + \beta)(\frac{\sigma^2_\theta(Var(S) - Cov(\hat{s}_i, S))}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}\hat{s}_i$$
$$+\frac{\sigma^2_\theta(Var(\hat{s}_i) - Cov(\hat{s}_i, S))}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}S + \eta_i - p).$$

For dealer $i+1$, market clearing condition implies

$$Q^{i+1}_{i,i+1} + a^i_{i,i+1}\hat{s}_i + b^i_{i,i+1}p + c^i_{i,i+1}\eta_i + d^i_{i,i+1}S + \beta p = 0,$$

thus

$$p = -\frac{a^i_{i,i+1}\hat{s}_i + c^i_{i,i+1}\eta_i}{b^i_{i,i+1} + \beta} - \frac{d^i_{i,i+1}S}{b^i_{i,i+1} + \beta} - \frac{Q^{i+1}_{i,i+1}}{b^i_{i,i+1} + \beta} := I_{i+1} - \frac{d^i_{i,i+1}S}{b^i_{i,i+1} + \beta} + \lambda^{i+1}_{i,i+1}Q^{i+1}_{i,i+1}.$$

Dealer $i+1$'s optimization problem is

$$\max_{Q^{i+1}_{i,i+1}}(\mathbb{E}(\theta_{i+1}|I_{i+1}, \eta_{i+1}) - p)Q^{i+1}_{i,i+1} = (\mathbb{E}(\theta_{i+1}|I_{i+1}, \eta_{i+1}) - I_{i+1} + \frac{d^i_{i,i+1}S}{b^i_{i,i+1} + \beta} - \lambda^{i+1}_{i,i+1}Q^{i+1}_{i,i+1})Q^{i+1}_{i,i+1},$$

FOC implies

$$-2\lambda^{i+1}_{i,i+1}Q^{i+1}_{i,i+1} + \mathbb{E}(\theta_{i+1}|I_{i+1}, \eta_{i+1}) - I_{i+1} + \frac{d^i_{i,i+1}S}{b^i_{i,i+1} + \beta} = 0.$$

Using projection theorem,

$$\begin{pmatrix} \theta \\ \hat{s}_{i+1} = I_{i+1}(-\frac{b^i_{i,i+1}+\beta}{a^i_{i,i+1}}) \\ S \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_\theta & \sigma^2_\theta & \sigma^2_\theta \\ \sigma^2_\theta & Var(\hat{s}_{i+1}) & Cov(\hat{s}_{i+1}, S) \\ \sigma^2_\theta & Cov(\hat{s}_{i+1}, S) & Var(S) \end{pmatrix}\right],$$

$$\mathbb{E}(\theta_{i+1}|\hat{s}_{i+1}, S, \eta_i) = \frac{\sigma^2_\theta(Var(S) - Cov(\hat{s}_{i+1}, S))}{Var(\hat{s}_{i+1})Var(S) - Cov(\hat{s}_{i+1}, S)^2}I_{i+1}(-\frac{b^i_{i,i+1} + \beta}{a^i_{i,i+1}})$$

$$+\frac{\sigma_\theta^2(Var(\hat{s}_{i+1})-Cov(\hat{s}_{i+1},S))}{Var(\hat{s}_{i+1})Var(S)-Cov(\hat{s}_{i+1},S)^2}S+\eta_{i+1}$$
$$=k_1I_{i+1}+k_2S+\eta_{i+1},$$

thus

$$-2\lambda_{i,i+1}^{i+1}Q_{i,i+1}^{i+1}+k_1I_{i+1}+k_2S+\eta_{i+1}-I_{i+1}+\frac{d_{i,i+1}^iS}{b_{i,i+1}^i+\beta}=0,$$

$$-2\lambda_{i,i+1}^{i+1}Q_{i,i+1}^{i+1}+(k_1-1)(p-\lambda_{i,i+1}^{i+1}Q_{i,i+1}^{i+1}+\frac{d_{i,i+1}^iS}{b_{i,i+1}^i+\beta})+k_2S+\frac{d_{i,i+1}^iS}{b_{i,i+1}^i+\beta}+\eta_{i+1}=0,$$

$$Q_{i,i+1}^{i+1}(-\lambda_{i,i+1}^{i+1})(1+k_1)+(k_1-1)p+k_2S+k_1\frac{d_{i,i+1}^iS}{b_{i,i+1}^i+\beta}+\eta_{i+1}=0,$$

we have

$$Q_{i,i+1}^{i+1}=-(b_{i,i+1}^i+\beta)(\frac{k_1-1}{k_1+1}p+\frac{k_2S+k_1\frac{d_{i,i+1}^i}{b_{i,i+1}^i+\beta}S+\eta_{i+1}}{k_1+1}),$$

thus using

$$b_{i,i+1}^i=b_{i,i+1}^{i+1}+\beta$$
$$b_{i,i+1}^{i+1}=-(b_{i,i+1}^i+\beta)\frac{k_1-1}{k_1+1},$$

we have

$$b_{i,i+1}^i=b_{i,i+1}^{i+1}+\beta=\frac{\beta}{k_1},$$
$$c_{i,i+1}^i=c_{i,i+1}^{i+1}=-\frac{\beta}{k_1}.$$

We also have

$$k_1=\frac{\sigma_\theta^2(Var(S)-Cov(\hat{s}_{i+1},S)}{Var(\hat{s}_{i+1})Var(S)-Cov(\hat{s}_{i+1},S)^2}(-\frac{b_{i,i+1}^i+\beta}{a_{i,i+1}^i})$$

$$=\frac{\sigma_\theta^2(Var(S)-Cov(\hat{s}_{i+1},S)}{Var(\hat{s}_{i+1})Var(S)-Cov(\hat{s}_{i+1},S)^2}(-\frac{b_{i,i+1}^{i+1}+2\beta}{-(b_{i,i+1}^{i+1}+\beta)\frac{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}})$$

$$=\frac{b_{i,i+1}^{i+1}+2\beta}{b_{i,i+1}^{i+1}+\beta}\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_{i+1},S)^2}{Var(\hat{s}_{i+1})Var(S)-Cov(\hat{s}_{i+1},S)^2},$$

thus

$$\frac{1}{k_1}=\frac{b_{i,i+1}^{i+1}+\beta}{b_{i,i+1}^{i+1}+2\beta}\frac{Var(\hat{s}_{i+1})Var(S)-Cov(\hat{s}_{i+1},S)^2}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_{i+1},S)^2},$$

using

$$Var(\hat{s}_{i+1})=Var(\hat{s}_i)+(\frac{c_{i,i+1}^i}{a_{i,i+1}^i})^2\sigma_\eta^2,$$

$$\frac{c_{i,i+1}^i}{a_{i,i+1}^i}=\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))},$$

we have

$$\frac{1}{k_1}=\frac{b_{i,i+1}^{i+1}+\beta}{b_{i,i+1}^{i+1}+2\beta}(1+(\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))})^2\sigma_\eta^2\frac{Var(S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_{i+1},S)^2}),$$

we have

$$b_{i,i+1}^{i+1}+\beta=\frac{b_{i,i+1}^{i+1}+\beta}{b_{i,i+1}^{i+1}+2\beta}(1+(\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))})^2\sigma_\eta^2\frac{Var(S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_{i+1},S)^2})\beta,$$

$$b_{i,i+1}^{i+1}+2\beta=\beta(1+\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma_\theta^4(Var(S)-Cov(\hat{s}_i,S))^2}\sigma_\eta^2Var(S)),$$

$$b_{i,i+1}^{i+1}+\beta=\beta\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma_\theta^4(Var(S)-Cov(\hat{s}_i,S))^2}\sigma_\eta^2Var(S),$$

using

$$Var(\theta|S)=\sigma_\theta^2-\frac{\sigma_\theta^4}{Var(S)},$$

$$Var(\theta|\hat{s}_i,S)=\sigma_\theta^2-\sigma_\theta^4\frac{Var(S)-2Cov(\hat{s}_i,S)+Var(\hat{s}_i)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2},$$

we have

$$Var(\theta|S)-Var(\theta|\hat{s}_i,S)=\sigma_\theta^4\frac{(Var(S)-Cov(\hat{s}_i,S))^2}{(Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2)Var(S)},$$

thus

$$b_{i,i+1}^{i+1}+\beta=\frac{1}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}\sigma_\eta^2\beta,$$

thus

$$k_1=\frac{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}{\sigma_\eta^2}.$$

Finally, we can solve

$$d_{i,i+1}^i=-\frac{\beta}{k_1}\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma_\theta^2=-\beta\frac{\sigma_\eta^2}{\sigma_\theta^2}\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{(Var(S)-Cov(\hat{s}_i,S))^2}Var(S),$$

$$d_{i,i+1}^{i+1}=-\beta(1+\frac{1}{k_1})(\frac{k_1\frac{d_{i,i+1}^i}{b_{i,i+1}^i+\beta}+k_2}{k_1+1})=-\beta(1+\frac{1}{k_1})(\frac{k_1\frac{-\frac{\beta}{k_1}\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma_\theta^2}{\beta(\frac{1}{k_1}+1)}+k_2}{k_1+1})$$

$$=-\frac{\beta}{k_1}(k_1\frac{-\frac{\beta}{k_1}\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma_\theta^2}{\beta(\frac{1}{k_1}+1)}+k_2)=-\frac{\beta}{k_1}(\frac{-\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma_\theta^2}{(\frac{1}{k_1}+1)}+k_2)$$

$$=\frac{\beta}{1+k_1}\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma_\theta^2-\frac{k_2}{k_1}\beta$$

$$=\frac{\beta}{1+k_1}\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma_\theta^2-\frac{\beta}{k_1}\frac{\sigma_\theta^2(Var(\hat{s}_{i+1})-Cov(\hat{s}_{i+1},S))}{Var(\hat{s}_{i+1})Var(S)-Cov(\hat{s}_{i+1},S)^2}$$

$$= \frac{\beta}{1+k_1}\frac{Var(\hat{s}_i) - Cov(\hat{s}_i, S)}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}\sigma_\theta^2$$

$$-\frac{\beta}{k_1}\frac{\sigma_\theta^2(Var(\hat{s}_i) + (\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))})^2\sigma_\eta^2 - Cov(\hat{s}_i, S))}{Var(\hat{s}_i)Var(S) + (\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))})^2\sigma_\eta^2 Var(S) - Cov(\hat{s}_i, S)^2}$$

$$= \frac{\beta}{1+k_1}\frac{Var(\hat{s}_i) - Cov(\hat{s}_i, S)}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}\sigma_\theta^2$$

$$-\frac{\beta}{k_1}\frac{\sigma_\theta^2(Var(\hat{s}_i) + (\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))})^2\sigma_\eta^2 - Cov(\hat{s}_i, S))}{(Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2)(1 + (\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{(\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S)))^2}\sigma_\eta^2)}$$

$$= \frac{\beta}{1+k_1}\frac{Var(\hat{s}_i) - Cov(\hat{s}_i, S)}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}\sigma_\theta^2$$

$$-\frac{\beta}{k_1}\frac{\sigma_\theta^2(Var(\hat{s}_i) + (\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))})^2\sigma_\eta^2 - Cov(\hat{s}_i, S))}{(Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2)(1 + \frac{1}{k_1})}$$

$$= \frac{\beta}{1+k_1}\frac{1}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}\sigma_\theta^2\Big(Var(\hat{s}_i) - Cov(\hat{s}_i, S) - Var(\hat{s}_i)$$

$$-(\frac{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}{\sigma_\theta^2(Var(S) - Cov(\hat{s}_i, S))})^2\sigma_\eta^2 + Cov(\hat{s}_i, S))\Big)$$

$$= \frac{\beta}{1+k_1}\frac{1}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}\sigma_\theta^2\Big(-(\frac{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}{\sigma_\theta^2(Var(S) - Cov(\hat{s}_i, S))})^2\sigma_\eta^2)\Big)$$

$$= -\frac{\beta}{1+k_1}\frac{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}{(Var(S) - Cov(\hat{s}_i, S))^2}\varphi = -\frac{\beta}{1+k_1}\frac{\sigma_\theta^4}{(Var(\theta|S) - Var(\theta|\hat{s}_i, S))Var(S)}\varphi$$

$$= -\frac{\beta}{1 + \frac{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}{\sigma_\eta^2}}\frac{\sigma_\theta^4}{(Var(\theta|S) - Var(\theta|\hat{s}_i, S))Var(S)}\varphi.$$

■

Proof of Lemma 5

Proof.

With public signal

$$\frac{c_{i,i+1}^i}{a_{i,i+1}^i} = \frac{\sigma_\theta^2}{Var(\theta|S) - Var(\theta|\hat{s}_i, S)}(1 - \frac{Cov(\hat{s}_i, S)}{Var(S)}),$$

$$Var(\theta|S) = \sigma_\theta^2 - \frac{\sigma_\theta^4}{Var(S)}.$$

Using projection theorem,

$$\begin{pmatrix} \theta \\ \hat{s}_{i+1} = \hat{s}_i + \frac{c^i_{i,i+1}}{a^i_{i,i+1}}\eta_i \\ S \end{pmatrix}$$

$$\sim \mathcal{N}\left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 & \sigma_\theta^2 \\ \sigma_\theta^2 & Var(\hat{s}_i) + (\frac{\sigma_\theta^2}{Var(\theta|S) - Var(\theta|\hat{s}_i, S)}(1 - \frac{Cov(\hat{s}_i, S)}{Var(S)}))^2\sigma_\eta^2 & Cov(\hat{s}_i, S) \\ \sigma_\theta^2 & Cov(\hat{s}_i, S) & Var(S) \end{pmatrix}\right],$$

we have

$$Var(\theta|\hat{s}_{i+1}, S)$$

$$= \sigma_\theta^2 - \frac{\begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 \end{pmatrix}\begin{pmatrix} Var(S) & -Cov(\hat{s}_i, S) \\ -Cov(\hat{s}_i, S) & Var(\hat{s}_i) + (\frac{\sigma_\theta^2}{Var(\theta|S) - Var(\theta|\hat{s}_i, S)}(1 - \frac{Cov(\hat{s}_i, S)}{Var(S)}))^2\sigma_\eta^2 \end{pmatrix}\begin{pmatrix} \sigma_\theta^2 \\ \sigma_\theta^2 \end{pmatrix}}{(Var(\hat{s}_i) + (\frac{\sigma_\theta^2}{Var(\theta|S) - Var(\theta|\hat{s}_i, S)}(1 - \frac{Cov(\hat{s}_i, S)}{Var(S)}))^2\sigma_\eta^2)Var(S) - Cov(\hat{s}_i, S)^2}$$

$$= \sigma_\theta^2 - \sigma_\theta^4 \frac{Var(S) + Var(\hat{s}_i) + (\frac{\sigma_\theta^2}{Var(\theta|S) - Var(\theta|\hat{s}_i, S)}(1 - \frac{Cov(\hat{s}_i, S)}{Var(S)}))^2\sigma_\eta^2 - 2Cov(\hat{s}_i, S)}{(Var(\hat{s}_i) + (\frac{\sigma_\theta^2}{Var(\theta|S) - Var(\theta|\hat{s}_i, S)}(1 - \frac{Cov(\hat{s}_i, S)}{Var(S)}))^2\sigma_\eta^2)Var(S) - Cov(\hat{s}_i, S)^2}.$$

Similarly,

$$\begin{pmatrix} \theta \\ \hat{s}_i \\ S \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 & \sigma_\theta^2 \\ \sigma_\theta^2 & Var(\hat{s}_i) & Cov(\hat{s}_i, S) \\ \sigma_\theta^2 & Cov(\hat{s}_i, S) & Var(S) \end{pmatrix}\right],$$

we have

$$Var(\theta|\hat{s}_i, S) = \sigma_\theta^2 - \sigma_\theta^4 \frac{Var(S) + Var(\hat{s}_i) - 2Cov(\hat{s}_i, S)}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}.$$

Define

$$\Delta V \equiv Var(\theta|S) - Var(\theta|\hat{s}_i, S) = -\frac{\sigma_\theta^4}{Var(S)} + \sigma_\theta^4 \frac{Var(S) + Var(\hat{s}_i) - 2Cov(\hat{s}_i, S)}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2},$$

we have

$$\frac{1}{Var(\theta|S) - Var(\theta|\hat{s}_{i+1}, S)}$$

$$= \frac{1}{\sigma_\theta^4} \frac{1}{\frac{Var(S) + Var(\hat{s}_i) + (\frac{\sigma_\theta^2}{Var(\theta|S) - Var(\theta|\hat{s}_i, S)}(1 - \frac{Cov(\hat{s}_i, S)}{Var(S)}))^2\sigma_\eta^2 - 2Cov(\hat{s}_i, S)}{(Var(\hat{s}_i) + (\frac{\sigma_\theta^2}{Var(\theta|S) - Var(\theta|\hat{s}_i, S)}(1 - \frac{Cov(\hat{s}_i, S)}{Var(S)}))^2\sigma_\eta^2)Var(S) - Cov(\hat{s}_i, S)^2} - \frac{1}{Var(S)}}$$

$$= \frac{1}{\sigma_\theta^4} \frac{(Var(\hat{s}_i) + (\frac{\sigma_\theta^2}{\Delta V}(1 - \frac{Cov(\hat{s}_i, S)}{Var(S)}))^2\sigma_\eta^2)Var(S) - Cov(\hat{s}_i, S)^2}{Var(S) + Var(\hat{s}_i) + (\frac{\sigma_\theta^2}{\Delta V}(1 - \frac{Cov(\hat{s}_i, S)}{Var(S)}))^2\sigma_\eta^2 - 2Cov(\hat{s}_i, S) - \frac{(Var(\hat{s}_i) + (\frac{\sigma_\theta^2}{\Delta V}(1 - \frac{Cov(\hat{s}_i, S)}{Var(S)}))^2\sigma_\eta^2)Var(S) - Cov(\hat{s}_i, S)^2}{Var(S)}}$$

$$= \frac{1}{\sigma_\theta^4} \frac{(Var(\hat{s}_i) + (\frac{\sigma_\theta^2}{\Delta V}(1 - \frac{Cov(\hat{s}_i,S)}{Var(S)}))^2\sigma_\eta^2)Var(S) - Cov(\hat{s}_i,S)^2}{Var(S) + Var(\hat{s}_i) + (\frac{\sigma_\theta^2}{\Delta V}(1 - \frac{Cov(\hat{s}_i,S)}{Var(S)}))^2\sigma_\eta^2 - 2Cov(\hat{s}_i,S) - (Var(\hat{s}_i) + (\frac{\sigma_\theta^2}{\Delta V}(1 - \frac{Cov(\hat{s}_i,S)}{Var(S)}))^2\sigma_\eta^2) + \frac{Cov(\hat{s}_i,S)^2}{Var(S)}}$$

$$= \frac{1}{\sigma_\theta^4} \frac{(Var(\hat{s}_i) + (\frac{\sigma_\theta^2}{\Delta V}(1 - \frac{Cov(\hat{s}_i,S)}{Var(S)}))^2\sigma_\eta^2)Var(S) - Cov(\hat{s}_i,S)^2}{Var(S) - 2Cov(\hat{s}_i,S) + \frac{Cov(\hat{s}_i,S)^2}{Var(S)}},$$

we have

$$\frac{\sigma_\eta^2}{Var(\theta|S) - Var(\theta|\hat{s}_{i+1},S)} - \frac{\sigma_\eta^2}{\Delta V} - (\frac{\sigma_\eta^2}{\Delta V})^2$$

$$= \sigma_\eta^2(\frac{1}{\sigma_\theta^4}\big((Var(\hat{s}_i) + (\frac{\sigma_\theta^2}{\Delta V}(1 - \frac{Cov(\hat{s}_i,S)}{Var(S)}))^2\sigma_\eta^2)Var(S) - Cov(\hat{s}_i,S)^2\big)\Delta V^2$$

$$-\sigma_\eta^2(Var(S) - 2Cov(\hat{s}_i,S) + \frac{Cov(\hat{s}_i,S)^2}{Var(S)}) - \Delta V(Var(S) - 2Cov(\hat{s}_i,S) + \frac{Cov(\hat{s}_i,S)^2}{Var(S)}))$$

$$\frac{1}{(Var(S) - 2Cov(\hat{s}_i,S) + \frac{Cov(\hat{s}_i,S)^2}{Var(S)})\Delta V^2}$$

$$= \sigma_\eta^2\Delta V \frac{(Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i,S)^2)(-\frac{1}{Var(S)} + \frac{Var(S)+Var(\hat{s}_i)-2Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}) - \frac{(Var(S)-Cov(\hat{s}_i,S))^2}{Var(S)}}{(Var(S) - 2Cov(\hat{s}_i,S) + \frac{Cov(\hat{s}_i,S)^2}{Var(S)})\Delta V^2}$$

$$= \sigma_\eta^2\Delta V \frac{\frac{Cov(\hat{s}_i,S)^2 - Var(\hat{s}_i)Var(S) - Var(S)^2 - Cov(\hat{s}_i,S)^2 + 2Var(S)Cov(\hat{s}_i,S)}{Var(S)} + Var(S) + Var(\hat{s}_i) - 2Cov(\hat{s}_i,S)}{(Var(S) - 2Cov(\hat{s}_i,S) + \frac{Cov(\hat{s}_i,S)^2}{Var(S)})\Delta V^2}$$

$$= \sigma_\eta^2\Delta V \frac{-Var(\hat{s}_i) - Var(S) + 2Cov(\hat{s}_i,S) + Var(S) + Var(\hat{s}_i) - 2Cov(\hat{s}_i,S)}{(Var(S) - 2Cov(\hat{s}_i,S) + \frac{Cov(\hat{s}_i,S)^2}{Var(S)})\Delta V^2} = 0.$$

■

Proof of Lemma 6

Proof.

Using the result

$$b_{i,i+1}^i = b_{i,i+1}^{i+1} + \beta = \beta\frac{\sigma_\eta^2}{Var(\theta|S) - Var(\theta|\hat{s}_i,S)},$$

$$c_{i,i+1}^i = c_{i,i+1}^{i+1} = -\beta\frac{\sigma_\eta^2}{Var(\theta|S) - Var(\theta|\hat{s}_i,S)},$$

$$\frac{c_{i,i+1}^i}{a_{i,i+1}^i} = \frac{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i,S)^2}{\sigma_\theta^2(Var(S) - Cov(\hat{s}_i,S))},$$

$$d_{i,i+1}^i = -\beta\frac{\sigma_\eta^2}{\sigma_\theta^2}\frac{Var(\hat{s}_i) - Cov(\hat{s}_i,S)}{(Var(S) - Cov(\hat{s}_i,S))^2}Var(S),$$

$$d_{i,i+1}^{i+1} = -\frac{\beta}{1 + \frac{Var(\theta|S) - Var(\theta|\hat{s}_i, S)}{\sigma_\eta^2}} \frac{\sigma_\theta^4}{(Var(\theta|S) - Var(\theta|\hat{s}_i, S))Var(S)}\varphi,$$

we have

$$p = -\frac{a_{i,i+1}^i}{b_{i,i+1}^i + b_{i,i+1}^{i+1} + \beta}\hat{s}_i - \frac{c_{i,i+1}^i \eta_i + c_{i,i+1}^{i+1}\eta_{i+1}}{b_{i,i+1}^i + b_{i,i+1}^{i+1} + \beta} - \frac{d_{i,i+1}^i + d_{i,i+1}^{i+1}}{b_{i,i+1}^i + b_{i,i+1}^{i+1} + \beta}S$$

$$= \frac{Var(\theta|S) - Var(\theta|\hat{s}_i, S)}{2\sigma_\theta^2(1 - \frac{Cov(\hat{s}_i, S)}{Var(S)})}\hat{s}_i - \frac{d_1 + d_2}{2\sigma_\eta^2 \beta}(Var(\theta|S) - Var(\theta|\hat{s}_i, S))S + \frac{\eta_1 + \eta_2}{2},$$

thus

$$\mathbb{E}[Q_{i,i+1}^{i+1}(\eta_{i+1} - \eta_i)] = \mathbb{E}[(a_i\hat{s}_i + b_{i,i+1}^i p_{i,i+1} + c_{i,i+1}^i \eta_i + d_{i,i+1}^i S)(\eta_{i+1} - \eta_i)] = c_{i,i+1}^i \sigma_\eta^2$$

$$= -\beta\frac{\sigma_\eta^4}{Var(\theta|S) - Var(\theta|\hat{s}_i, S)},$$

$$\mathbb{E}[\beta p_{i,i+1}(p_{i,i+1} - \theta - \eta_i)] = \beta\Big(\frac{(Var(\theta|S) - Var(\theta|\hat{s}_i, S))^2 Var(\hat{s}_i)}{4\sigma_\theta^4(1 - \frac{Cov(\hat{s}_i, S)}{Var(S)})^2} - \frac{Var(\theta|S) - Var(\theta|\hat{s}_i, S)}{2(1 - \frac{Cov(\hat{s}_i, S)}{Var(S)})}$$

$$+ \frac{(d_{i,i+1}^i + d_{i,i+1}^{i+1})^2}{4\sigma_\eta^4\beta^2}(Var(\theta|S) - Var(\theta|\hat{s}_i, S))^2 Var(S) + \frac{d_{i,i+1}^i + d_{i,i+1}^{i+1}}{2\varphi\beta}(Var(\theta|S) - Var(\theta|\hat{s}_i, S))$$

$$- \frac{d_{i,i+1}^i + d_{i,i+1}^{i+1}}{2\sigma_\eta^2\sigma_\theta^2\beta(1 - \frac{Cov(\hat{s}_i, S)}{Var(S)})}(Var(\theta|S) - Var(\theta|\hat{s}_i, S))^2 Cov(\hat{s}_i, S)\Big).$$

■

Proof of Lemma 7

Proof.

Dealer n_p learns a private signal $s_{n_p} = \theta + \varepsilon_{n_p}$, where $\gamma_{n_p} \equiv \frac{\sigma_{n_p}^2}{\sigma_\theta^2}$.

Dealer n_p and n_p+1 observe a public signal $S = \theta + \varepsilon_S$, where $\varepsilon_S \sim \mathcal{N}(0, \sigma_p^2)$, $\varepsilon_{n_p} \perp \varepsilon_S$, $\gamma_p \equiv \frac{\sigma_p^2}{\sigma_\theta^2}$.

Without public signal,

$$Var(\theta|s_{n_p}) = \sigma_\theta^2 - \frac{\sigma_\theta^2}{1 + \gamma_{n_p}},$$

thus

$$Var(\theta) - Var(\theta|s_1) = \frac{\sigma_\theta^2}{1 + \gamma_{n_p}}.$$

With public signal,

$$\begin{pmatrix} \theta \\ s_{n_p} \\ S \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 & \sigma_\theta^2 \\ \sigma_\theta^2 & \sigma_\theta^2(1+\gamma_{n_p}) & \sigma_\theta^2 \\ \sigma_\theta^2 & \sigma_\theta^2 & \sigma_\theta^2(1+\gamma_p) \end{pmatrix}\right],$$

thus

$$Var(\theta|s_{n_p}, S) = \sigma_\theta^2 - \sigma_\theta^2 \frac{\gamma_{n_p} + \gamma_p}{(1+\gamma_{n_p})(1+\gamma_p) - 1},$$

$$Var(\theta|S) = \sigma_\theta^2 - \frac{\sigma_\theta^2}{1+\gamma_p},$$

we have

$$Var(\theta|S) - Var(\theta|s_{n_p}, S) = -\frac{\sigma_\theta^2}{1+\gamma_p} + \sigma_\theta^2 \frac{\gamma_{n_p} + \gamma_p}{(1+\gamma_{n_p})(1+\gamma_p) - 1},$$

thus the change of information asymmetry is

$$\begin{aligned} & Var(\theta|S) - Var(\theta|s_{n_p}, S) - (Var(\theta) - Var(\theta|s_{n_p})) \\ = & \frac{-2\gamma_{n_p}\gamma_p - \gamma_{n_p} - \gamma_p}{(1+\gamma_{n_p})(1+\gamma_p)((1+\gamma_{n_p})(1+\gamma_p) - 1)} \sigma_\theta^2 < 0. \end{aligned}$$

■

Proof of Lemma 8

Proof.

Using the results in Lemma 1.6, dealers' profit from asset reallocation is

$$\mathbb{E}(Q_{i,i+1}^{i+1}(\eta_{i+1} - \eta_i)) = \frac{-\beta\sigma_\eta^4}{Var(\theta|S) - Var(\theta|s_{n_p}, S)},$$

thus the increase of profit from asset reallocation is

$$-\beta\sigma_\eta^4 \frac{1}{Var(\theta|S) - Var(\theta|s_{n_p}, S)} + \beta\sigma_\eta^4 \frac{1}{Var(\theta) - Var(\theta|s_{n_p})} > 0,$$

we have $\lim_{\gamma_p \to 0} \frac{1}{Var(\theta|S) - Var(\theta|s_{n_p}, S)} = \infty$.

From the results in Proposition 2, we have

$$\frac{c_{i,i+1}^i}{b_{i,i+1}^i + b_{i,i+1}^{i+1} + \beta} = \frac{c_{i,i+1}^{i+1}}{b_{i,i+1}^i + b_{i,i+1}^{i+1} + \beta} = -\frac{1}{2},$$

$$\frac{a^i_{i,i+1}}{b^i_{i,i+1}+b^{i+1}_{i,i+1}+\beta} = -\frac{1}{2}\frac{\gamma_p}{(1+\gamma)(1+\gamma_p)-1},$$

$$\frac{d^i_{i,i+1}}{b^i_{i,i+1}+b^{i+1}_{i,i+1}+\beta} = -\frac{1}{2}\frac{\gamma}{(1+\gamma)(1+\gamma_p)-1},$$

$$\frac{d^{i+1}_{i,i+1}}{b^i_{i,i+1}+b^{i+1}_{i,i+1}+\beta} = -\frac{1}{2}\frac{((1+\gamma_{n_p})(1+\gamma_p)-1)^2\varphi}{((1+\gamma_{n_p})\gamma_p^2+((1+\gamma_{n_p})(1+\gamma_p)-1)^2\varphi)(1+\gamma_p)-\gamma_p^2},$$

we have

$$\lim_{\gamma_p\to 0}\frac{a^i_{i,i+1}}{b^i_{i,i+1}+b^{i+1}_{i,i+1}+\beta} = 0,$$

$$\lim_{\gamma_p\to 0}\frac{d^i_{i,i+1}}{b^i_{i,i+1}+b^{i+1}_{i,i+1}+\beta} = \lim_{\gamma_p\to 0}\frac{d^{i+1}_{i,i+1}}{b^i_{i,i+1}+b^{i+1}_{i,i+1}+\beta} = -\frac{1}{2},$$

thus

$$\lim_{\gamma_p\to 0} p = \frac{\eta_i+\eta_{i+1}}{2}+\frac{S}{2},$$

thus

$$\lim_{\gamma_p\to 0}\beta\mathbb{E}(p^2-p\theta-p\eta_i) = \beta(\frac{\sigma_\eta^2}{2}+\frac{\sigma_\theta^2}{4}-\frac{\sigma_\theta^2}{2}-\frac{\sigma_\eta^2}{2}) = -\beta\frac{\sigma_\theta^2}{4},$$

thus we have if γ_p is relatively small, dealers' profit $\mathbb{E}(Q^{i+1}_{i,i+1}(\eta_{i+1}-\eta_i))+\beta\mathbb{E}(p^2-p\theta-p\eta_i)$ is larger in the case with public signal.

■

Proof of Proposition 3

Proof.

(1) $n_p = 3$

In the model with transparency, what is public is

$$S \equiv 2(1+\tau)p_{1,2} = s_1+(1+\tau)(\eta_1+\eta_2) = s_1+\frac{\kappa_1}{\varphi}(\eta_1+\eta_2),$$

from trading with dealer 2, dealer 3 observes $s_3 = s_1+\frac{\kappa_1}{\varphi}\eta_1+\frac{\kappa_2}{\varphi}\eta_2$.

Without public signal,

$$Var(\theta|s_3) = \sigma_\theta^2-\frac{\sigma_\theta^2}{\kappa_3}\varphi,$$

thus

$$Var(\theta)-Var(\theta|s_3) = \frac{\sigma_\theta^2}{\kappa_3}\varphi.$$

With public signal, it's equivalent to dealer 3 observes $s_1 + \frac{\kappa_1}{\varphi}\eta_1$,

$$\begin{pmatrix} \theta \\ s_1 + \frac{\kappa_1}{\varphi}\eta_1 \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 \\ \sigma_\theta^2 & \frac{\sigma_\eta^2}{\varphi^2}\kappa_2 \end{pmatrix}\right],$$

$$\begin{pmatrix} \theta \\ S \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 \\ \sigma_\theta^2 & \frac{\sigma_\eta^2}{\varphi^2}\kappa_1 + 2\frac{\sigma_\eta^2}{\varphi^2}\kappa_1^2 \end{pmatrix}\right],$$

thus

$$Var(\theta|s_1 + \frac{\kappa_1}{\varphi}\eta_1) = \sigma_\theta^2 - \frac{\sigma_\theta^2}{\kappa_2}\varphi,$$

$$Var(\theta|S) = \sigma_\theta^2 - \frac{\sigma_\theta^2}{\kappa_1 + 2\kappa_1^2}\varphi,$$

we have

$$Var(\theta|S) - Var(\theta|s_1 + \frac{\kappa_1}{\varphi}\eta_1) = -\frac{\sigma_\theta^2}{\kappa_1 + 2\kappa_1^2}\varphi + \frac{\sigma_\theta^2}{\kappa_2}\varphi,$$

thus the change of information asymmetry is

$$Var(\theta|S) - Var(\theta|s_1 + \frac{\kappa_1}{\varphi}\eta_1) - (Var(\theta) - Var(\theta|s_3)) = -\frac{\sigma_\theta^2}{\kappa_1 + 2\kappa_1^2}\varphi + \frac{\sigma_\theta^2}{\kappa_2}\varphi - \frac{\sigma_\theta^2}{\kappa_3}\varphi$$

$$= \sigma_\theta^2\varphi(\frac{1}{\kappa_2} - \frac{1}{\kappa_3} - \frac{1}{\kappa_2 + \kappa_1^2}) = \sigma_\theta^2\varphi\frac{\kappa_2(\kappa_1^2 - 1)}{(\kappa_2 + \kappa_2^2)(\kappa_2 + \kappa_1^2)}.$$

we have

$$\mathbb{E}(Q_{3,4}^4(\eta_4 - \eta_3)) = -\beta\sigma_\eta^2(\frac{\kappa_2^2}{\kappa_1^2} + \kappa_2) = -\beta\sigma_\eta^2\varphi\sigma_\theta^2\frac{1}{Var(\theta|S) - Var(\theta|s_1 + \frac{\kappa_1}{\varphi}\eta_1)},$$

thus the increase of profit from asset reallocation is

$$-\beta\sigma_\eta^4\frac{1}{Var(\theta|S) - Var(\theta|s_1 + \frac{\kappa_1}{\varphi}\eta_1)} + \beta\sigma_\eta^4\frac{1}{Var(\theta) - Var(\theta|s_3)} > 0 \quad \text{iff} \quad \kappa_1 < 1.$$

(2) $n_p > 3$

Without public signal,

$$Var(\theta|s_n) = \sigma_\theta^2 - \frac{\sigma_\theta^2}{\kappa_n}\varphi$$

thus

$$Var(\theta) - Var(\theta|s_n) = \frac{\sigma_\theta^2}{\kappa_n}\varphi.$$

with public signal

$$\begin{pmatrix} \theta \\ s_n \\ S \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 & \sigma_\theta^2 \\ \sigma_\theta^2 & \frac{\sigma_\eta^2}{\varphi^2}\kappa_n & \frac{\sigma_\eta^2}{\varphi^2}(\kappa_2 + \kappa_1\kappa_2) \\ \sigma_\theta^2 & \frac{\sigma_\eta^2}{\varphi^2}(\kappa_2 + \kappa_1\kappa_2) & \frac{\sigma_\eta^2}{\varphi^2}(\kappa_2 + \kappa_1^2) \end{pmatrix}\right],$$

thus

$$Var(\theta|s_n,S)=\sigma_\theta^2-\varphi\frac{(\kappa_1^2-\kappa_1\kappa_2)\sigma_\theta^2+(\kappa_n-\kappa_2(1+\kappa_1))\sigma_\theta^2}{\kappa_n(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2},$$

we have

$$Var(\theta|S)-Var(\theta|s_n,S)=-\frac{\sigma_\theta^2}{\kappa_1+2\kappa_1^2}\varphi+\varphi\frac{(\kappa_1^2-\kappa_1\kappa_2)\sigma_\theta^2+(\kappa_n-\kappa_2(1+\kappa_1))\sigma_\theta^2}{\kappa_n(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2},$$

thus the change of information asymmetry is

$$Var(\theta|S)-Var(\theta|s_n,S)-(Var(\theta)-Var(\theta|s_n))=$$

$$-\frac{\sigma_\theta^2}{\kappa_1+2\kappa_1^2}\varphi+\varphi\frac{(\kappa_1^2-\kappa_1\kappa_2)\sigma_\theta^2+(\kappa_n-\kappa_2(1+\kappa_1))\sigma_\theta^2}{\kappa_n(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2}-\frac{\sigma_\theta^2}{\kappa_n}\varphi.$$

Define

$$M\equiv\frac{\kappa_n(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2}{(\kappa_n+(\frac{c_n}{a_n})^2\varphi^2)(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2},$$

where $\frac{c^n_{n,n+1}}{a^n_{n,n+1}}=\frac{1}{\varphi}\frac{\kappa_n(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2}{\kappa_1^2-\kappa_1\kappa_2}$.

We have

$$\mathbb{E}(Q^{n+1}_{n,n+1}(\eta_{n+1}-\eta_n))=c_{n+1}\sigma_\eta^2$$

$$=\beta(1-\frac{1}{M})\sigma_\eta^2=\beta(1-\frac{(\kappa_n+(\frac{c_n}{a_n})^2\varphi^2)(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2}{\kappa_n(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2})\sigma_\eta^2$$

$$=\beta(1-\frac{(\kappa_n+(\frac{\kappa_n(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2}{\kappa_1^2-\kappa_1\kappa_2})^2)(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2}{\kappa_n(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2})\sigma_\eta^2$$

$$=-\beta\frac{(\frac{\kappa_n(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2}{\kappa_1^2-\kappa_1\kappa_2})^2(\kappa_2+\kappa_1^2)}{\kappa_n(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2}\sigma_\eta^2=-\beta\frac{\kappa_n(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2}{(\kappa_1^2-\kappa_1\kappa_2)^2}(\kappa_2+\kappa_1^2)\sigma_\eta^2,$$

as

$$Var(\theta|S)-Var(\theta|s_n,S)=\sigma_\theta^2\varphi\frac{(\kappa_1^2-\kappa_1\kappa_2)^2}{(\kappa_1+2\kappa_1^2)(\kappa_n(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2)},$$

thus

$$\mathbb{E}(Q^{n+1}_{n,n+1}(\eta_{n+1}-\eta_n))=-\beta\sigma_\eta^4\frac{1}{Var(\theta|S)-Var(\theta|s_n,S)},$$

as

$$\frac{1}{Var(\theta|S)-Var(\theta|s_n,S)}-\frac{1}{Var(\theta)-Var(\theta|s_n)}$$

$$=\sigma_\theta^2\varphi(\frac{\kappa_n(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2}{(\kappa_1^2-\kappa_1\kappa_2)^2}(\kappa_2+\kappa_1^2)-\kappa_n)$$

$$=\sigma_\theta^2\varphi\frac{\kappa_n((\kappa_2+\kappa_1^2)^2-(\kappa_1^2-\kappa_1\kappa_2)^2)-(\kappa_2+\kappa_1\kappa_2)^2(\kappa_2+\kappa_1^2)}{(\kappa_1^2-\kappa_1\kappa_2)^2}$$

$$= \sigma_\theta^2 \varphi \frac{\kappa_n(\kappa_2^2 + \kappa_1^4 + 2\kappa_1^2\kappa_2 - (\kappa_1^4 + \kappa_1^2\kappa_2^2 - 2\kappa_1^3\kappa_2)) - (\kappa_1 + \kappa_1\kappa_2)^2(\kappa_2 + \kappa_1^2)}{(\kappa_1^2 - \kappa_1\kappa_2)^2}$$

$$= \sigma_\theta^2 \varphi \frac{\kappa_n((\kappa_1 + \kappa_1^2)^2 + 2\kappa_1^2(\kappa_1 + \kappa_1^2) - \kappa_1^2(\kappa_1 + \kappa_1^2)^2 + 2\kappa_1^3(\kappa_1 + \kappa_1^2)) - (\kappa_1 + \kappa_1\kappa_2)^2(\kappa_2 + \kappa_1^2)}{(\kappa_1^2 - \kappa_1\kappa_2)^2}$$

$$= \sigma_\theta^2 \varphi \frac{\kappa_n(\kappa_1 + \kappa_1^2)(\kappa_1 + \kappa_1^2 + 2\kappa_1^2 - \kappa_1^3 - \kappa_1^4 + 2\kappa_1^3) - (\kappa_1 + \kappa_1\kappa_2)^2(\kappa_2 + \kappa_1^2)}{(\kappa_1^2 - \kappa_1\kappa_2)^2}$$

$$= \sigma_\theta^2 \varphi \frac{\kappa_n(\kappa_1 + \kappa_1^2)(\kappa_1 + 3\kappa_1^2 + \kappa_1^3 - \kappa_1^4) - (\kappa_1 + \kappa_1\kappa_2)^2(\kappa_2 + \kappa_1^2)}{(\kappa_1^2 - \kappa_1\kappa_2)^2}.$$

Obviously, it's negative if κ_1 is large enough.

■

Proof of Proposition 4

Proof.

(1) $n_p = 3$

Dealers' profit from serving the clients

$$\mathbb{E}(\beta p(p - \theta - \eta_3)) = \beta\sigma_\eta^2(-\frac{1}{4\kappa_2} + \frac{1}{4}\frac{\kappa_2^2(\kappa_1^2 + \kappa_2)}{(\frac{\kappa_2^2}{\kappa_1^2} + \kappa_2 + 1)^2\kappa_1^4}),$$

we have the magnitude of the increase of dealers' profit from serving the clients

$$-\beta\sigma_\eta^2(\frac{1}{4\kappa_2} - \frac{1}{4}\frac{\kappa_2^2(\kappa_1^2 + \kappa_2)}{(\frac{\kappa_2^2}{\kappa_1^2} + \kappa_2 + 1)^2\kappa_1^4} - \frac{1}{4\kappa_3}),$$

is smaller than the increase of profit from asset reallocation between dealers when κ_1 is relatively large.

(2) $n_p > 3$

Dealers' profit from serving the clients

$$\mathbb{E}(\beta p(p - \theta - \eta_n)) = \beta(\frac{1}{4}A^2\sigma_\eta^2\kappa_n + \frac{1}{4}B^2\sigma_\eta^2(\kappa_2 + \kappa_1^2) + \frac{1}{2}AB\sigma_\eta^2(\kappa_2 + \kappa_1\kappa_2) - \frac{1}{2}A\sigma_\eta^2 - \frac{1}{2}B\sigma_\eta^2),$$

where

$$A = \frac{\kappa_1^2 - \kappa_1\kappa_2}{\kappa_n(\kappa_2 + \kappa_1^2) - (\kappa_2 + \kappa_1\kappa_2)^2},$$

$$B = \frac{\kappa_n - \kappa_2(1 + \kappa_1)}{\kappa_n(\kappa_2 + \kappa_1^2) - (\kappa_2 + \kappa_1\kappa_2)^2}$$

$$+ \frac{(\kappa_n(\kappa_2 + \kappa_1^2) - (\kappa_2 + \kappa_1\kappa_2)^2)^2}{(\kappa_n + (\frac{\kappa_n(\kappa_2 + \kappa_1^2) - (\kappa_2 + \kappa_1\kappa_2)^2}{\kappa_1^2 - \kappa_1\kappa_2})^2)(\kappa_2 + \kappa_1^2) - (\kappa_2 + \kappa_1\kappa_2)^2}\frac{1}{(\kappa_1^2 - \kappa_1\kappa_2)^2},$$

as κ_1 is relatively large, we have $\kappa_n \gg \kappa_1, \kappa_2$, thus $A \to \frac{\kappa_1^2 - \kappa_1\kappa_2}{\kappa_n(\kappa_1^2+\kappa_2)}$, $B \to \frac{2}{\kappa_2+\kappa_1^2}$.

We have the magnitude of the increase of dealers' profit from serving the clients

$$\beta\sigma_\eta^2\left(\frac{1}{4}A^2\kappa_n + \frac{1}{4}B^2(\kappa_2+\kappa_1^2) + \frac{1}{2}AB(\kappa_2+\kappa_1\kappa_2) - \frac{1}{2}A - \frac{1}{2}B + \frac{1}{4\kappa_3}\right),$$

is smaller than the increase of profit from asset reallocation between dealers when κ_1 is relatively large.

∎

Proof of Proposition 5

Proof.

(1) For $i \in \{n_p+1, ..., \bar{n}\}$, in equilibrium $\hat{l}_{i-1,i} = 1$, if and only if $C \leq C^*_{i-1,i}(\kappa_1)$, where

$$C^*_{i-1,i}(\kappa_1) \equiv \max\{C^*(i), ..., C^*(\bar{n})\}.$$

For $j \in \{i, ..., \bar{n}\}$,

$$C^*(j) = \min\{\tilde{C}_{1,2}, ..., \tilde{C}_{j-1,j}\}.$$

For $k \in \{2, ..., j\}$,

$$\tilde{C}_{k-1,k} = \frac{\mathbb{E}(\pi^{k-1}_{k-1,k}) + \mathbb{E}(\pi^{k}_{k-1,k}) + \frac{1}{2}\left(\mathbb{E}(\pi^{k}_{k,k+1}) + \mathbb{E}(\pi^{k+1}_{k,k+1})\right) + ...\frac{1}{2^{j-k}}\left(\mathbb{E}(\pi^{j-1}_{j-1,j}) + \mathbb{E}(\pi^{j}_{j-1,j})\right)}{1 + \frac{1}{2} + ...\frac{1}{2^{j-k}}}.$$

As $j \geq i \geq n_p+1$, from Proposition 4, we have when κ_1 is relatively large, $\mathbb{E}(\pi^{j-1}_{j-1,j}) + \mathbb{E}(\pi^{j}_{j-1,j})$ is strictly smaller in the model with $p_{1,2}$ as public signal. Thus $\tilde{C}_{k-1,k}$ is strictly smaller in the model with $p_{1,2}$ as public signal, for $k \in \{2, ..., j\}$. Thus $C^*(j)$ is strictly smaller in the model with $p_{1,2}$ as public signal, for $j \in \{i, ..., \bar{n}\}$. Thus $C^*_{i-1,i}(\kappa_1)$ is strictly smaller in the model with $p_{1,2}$ as public signal, for $i \in \{n_p+1, ..., \bar{n}\}$.

(2) For $i \in \{2, ..., n_p\}$, for $j \geq i \geq 2$, from Proposition 4, we have when κ_1 is relatively large, $\mathbb{E}(\pi^{j-1}_{j-1,j}) + \mathbb{E}(\pi^{j}_{j-1,j})$ is weakly smaller in the model with $p_{1,2}$ as public signal. Thus $\tilde{C}_{k-1,k}$ is weakly smaller in the model with $p_{1,2}$ as public signal, for $k \in \{2, ..., j\}$. Thus $C^*(j)$ is weakly smaller in the model with $p_{1,2}$ as public signal, for $j \in \{i, ..., \bar{n}\}$. Thus $C^*_{i-1,i}(\kappa_1)$ is weakly smaller in the model with $p_{1,2}$ as public signal, for $i \in \{2, ..., n_p\}$.

∎

Proof of Proposition 6

Proof.

Suppose there are n dealers in the network. For dealer $n-1$, market clearing condition for the link $(n-2, n-1)$ implies

$$Q^{n-1}_{n-2,n-1} + a_{n-2}s_{n-2} + b_{n-2}p_{n-2,n-1} + c_{n-2}\eta_{n-2} + \beta p_{n-2,n-1} = 0,$$

thus

$$p_{n-2,n-1} = -\frac{a_{n-2}s_{n-2} + c_{n-2}\eta_{n-2}}{b_{n-2}+\beta} - \frac{Q^{n-1}_{n-2,n-1}}{b_{n-2}+\beta} := I^{n-1}_{n-2,n-1} + \lambda_{n-1}Q^{n-1}_{n-2,n-1}.$$

Market clearing condition for the link $(n-1, n)$ implies

$$Q^{n-1}_{n-1,n} + b_n p_{n-1,n} + c_n\eta_n + \beta p_{n-1,n} = 0,$$

thus

$$p_{n-1,n} = -\frac{c_n\eta_n}{b_n+\beta} - \frac{Q^{n-1}_{n-1,n}}{b_n+\beta} := I^{n-1}_{n-1,n} + \lambda_{n-1}Q^{n-1}_{n-1,n}.$$

The optimization problem of dealer $n-1$ is

$$\begin{aligned}
\max_{Q^{n-1}_{n-2,n-1}, Q^{n-1}_{n-1,n}} & (\mathbb{E}(\theta_{n-1}|I^{n-1}_{n-2,n-1}, \eta_{n-1}) - p_{n-2,n-1})Q^{n-1}_{n-2,n-1} \\
& + (\mathbb{E}(\theta_{n-1}|I^{n-1}_{n-1,n}, I^{n-1}_{n-2,n-1}, \eta_{n-1}) - p_{n-1,n})Q^{n-1}_{n-1,n} \\
= & (\mathbb{E}(\theta_{n-1}|I^{n-1}_{n-2,n-1}, \eta_{n-1}) - I^{n-1}_{n-2,n-1} - \lambda_{n-1}Q^{n-1}_{n-2,n-1})Q^{n-1}_{n-2,n-1} \\
& + (\mathbb{E}(\theta_{n-1}|I^{n-1}_{n-2,n-1}, \eta_{n-1}) - I^{n-1}_{n-1,n} - \lambda_{n-1}Q^{n-1}_{n-1,n})Q^{n-1}_{n-1,n},
\end{aligned}$$

the above equality uses that $I^{n-1}_{n-1,n}$ is not informative about θ.

Thus it's immediately clear that $Q^{n-1}_{n-2,n-1}$ does not depend on $I^{n-1}_{n-1,n}$, thus does not depend on $p_{n-1,n}$. Going backward, we can see the demand function of any $i \in \{2, ..., n\}$ is $Q^i_{i-1,i}$ just depends on $p_{i-1,i}$ and η_i. Thus the solution to the linear equilibrium of the simultaneous trading game is equivalent to that of the sequential trading game.

■

Proof of Proposition 7

Proof.

Starting from an equilibrium in the OTC game, we construct a bidding strategy for dealer i as follows. For dealer i,

$$q^i_{i,i+1,0} = a^i_{i,i+1} s_i + b^i_{i,i+1} p^i_{i,i+1,0} + c^i_{i,i+1} \eta_i,$$

$$p^i_{i,i+1,0} = 0,$$

$$q^i_{i,i+1,\tau+1} = a^i_{i,i+1} s_i + b^i_{i,i+1} p^i_{i,i+1,\tau+1} + c^i_{i,i+1} \eta_i,$$

$$p^i_{i,i+1,\tau+1} = p^{i+1}_{i,i+1,\tau}.$$

For dealer $i+1$,

$$q^{i+1}_{i,i+1,0} = b^{i+1}_{i,i+1} p^{i+1}_{i,i+1,0} + c^{i+1}_{i,i+1} \eta_{i+1},$$

$$p^{i+1}_{i,i+1,0} = 0,$$

$$q^{i+1}_{i,i+1,\tau+1} = b^{i+1}_{i,i+1} p^i_{i,i+1,\tau+1} + c^{i+1}_{i,i+1} \eta_{i+1},$$

$$p^{i+1}_{i,i+1,\tau+1} = \frac{\varphi}{2\kappa_i} \frac{q^i_{i,i+1,\tau} - b^i_{i,i+1} p^i_{i,i+1,\tau}}{a^i_{i,i+1}} + \frac{\eta_{i+1}}{2}.$$

First, we show that if bidding functions are defined as above, the OTC price-discovery process converges to the equilibrium prices and quantities in the OTC game.

In round 1, given the bids in round 0,

$$q^i_{i,i+1,1} = a^i_{i,i+1} s_i + b^i_{i,i+1} p^i_{i,i+1,1} + c^i_{i,i+1} \eta_i,$$

$$p^i_{i,i+1,1} = 0,$$

$$q^{i+1}_{i,i+1,1} = b^{i+1}_{i,i+1} p^{i+1}_{i,i+1,1} + c^{i+1}_{i,i+1} \eta_i,$$

$$p^{i+1}_{i,i+1,1} = \frac{\varphi}{2\kappa_i}(s_i + \frac{\kappa_i}{\varphi}\eta_i) + \frac{\eta_{i+1}}{2}.$$

In round 2, given the bids in round 1,

$$q^i_{i,i+1,2} = a^i_{i,i+1} s_i + b^i_{i,i+1} p^i_{i,i+1,2} + c^i_{i,i+1} \eta_i,$$

$$p^i_{i,i+1,2} = \frac{\varphi}{2\kappa_i}(s_i + \frac{\kappa_i}{\varphi}\eta_i) + \frac{\eta_{i+1}}{2},$$

$$q^{i+1}_{i,i+1,2} = b^{i+1}_{i,i+1} p^{i+1}_{i,i+1,2} + c^{i+1}_{i,i+1} \eta_{i+1},$$

$$p^{i+1}_{i,i+1,2} = \frac{\varphi}{2\kappa_i}(s_i + \frac{\kappa_i}{\varphi}\eta_i) + \frac{\eta_{i+1}}{2},$$

the trade takes place.

Then I show that dealer i and $i+1$ would not want to change their bidding strategy unilaterally.

Firstly, I show that for dealer $i+1$, given dealer i's strategy, it does not have incentive to deviate. As given dealer i's bid in last round τ, dealer $i+1$ learns $s_i + \frac{\kappa_i}{\varphi}\eta_i$. It can set price $p^{i+1}_{i,i+1,\tau+1}$ such that the trade takes place in next round with $p^{i+1}_{i,i+1,\tau+1}$, $q^{i}_{i,i+1,\tau+2}$, $q^{i+1}_{i,i+1,\tau+2}$, such that

$$q^{i}_{i,i+1,\tau+2} = a^{i}_{i,i+1}s_i + b^{i}_{i,i+1}p^{i+1}_{i,i+1,\tau+1} + c^{i}_{i,i+1}\eta_i,$$

$$q^{i+1}_{i,i+1,\tau+2} + q^{i}_{i,i+1,\tau+2} + \beta p^{i+1}_{i,i+1,\tau+1} = 0,$$

thus dealer $i+1$ solves

$$\max_{p^{i+1}_{i,i+1,\tau+1}} \mathbb{E}[q^{i+1}_{i,i+1,\tau+2}(\theta + \eta_{i+1} - p^{i+1}_{i,i+1,\tau+1})|s_i + \frac{\kappa_i}{\varphi}\eta_i, \eta_{i+1}],$$

s.t.

$$q^{i+1}_{i,i+1,\tau+2} + a^{i}_{i,i+1}s_i + b^{i}_{i,i+1}p^{i+1}_{i,i+1,\tau+1} + c^{i}_{i,i+1}\eta_i + \beta p^{i+1}_{i,i+1,\tau+1} = 0,$$

by construction,

$$a^{i}_{i,i+1} = -\beta\varphi,$$

$$b^{i}_{i,i+1} = \beta\kappa_i,$$

$$c^{i}_{i,i+1} = -\beta\kappa_i,$$

thus

$$\mathbb{E}[q^{i+1}_{i,i+1,\tau+2}(\theta + \eta_{i+1} - p^{i+1}_{i,i+1,\tau+1})|s_i + \frac{\kappa_i}{\varphi}\eta_i, \eta_{i+1}]$$

$$= \mathbb{E}[-(a^{i}_{i,i+1}s_i + b^{i}_{i,i+1}p^{i+1}_{i,i+1,\tau+1} +^{i}_{i,i+1}\eta_i + \beta p^{i+1}_{i,i+1,\tau+1})(\theta + \eta_{i+1} - p^{i+1}_{i,i+1,\tau+1})|s_i + \frac{\kappa_i}{\varphi}\eta_i, \eta_{i+1}]$$

$$= \mathbb{E}[-(-\beta\varphi(s_i + \frac{\kappa_i}{\varphi}\eta_i) + \beta\kappa_i p^{i+1}_{i,i+1,\tau+1} + \beta p^{i+1}_{i,i+1,\tau+1})(\theta + \eta_{i+1} - p^{i+1}_{i,i+1,\tau+1})|s_i + \frac{\kappa_i}{\varphi}\eta_i, \eta_{i+1}],$$

thus dealer $i+1$ solves

$$\max_{p^{i+1}_{i,i+1,\tau+1}} \mathbb{E}[(p^{i+1}_{i,i+1,\tau+1})^2\beta(1+\kappa_i) + p^{i+1}_{i,i+1,\tau+1}(-\beta\varphi(s_i + \frac{\kappa_i}{\varphi}\eta_i) - \beta(1+\kappa_i)(\theta + \eta_{i+1}))|s_i + \frac{\kappa_i}{\varphi}\eta_i, \eta_{i+1}],$$

we have

$$p^{i+1}_{i,i+1,\tau+1} = -\frac{-\beta\varphi(s_i + \frac{\kappa_i}{\varphi}\eta_i) - \beta(1+\kappa_i)(\mathbb{E}(\theta|s_i + \frac{\kappa_i}{\varphi}\eta_i) + \eta_{i+1})}{2\beta(1+\kappa_i)}$$

$$= \frac{\varphi(s_i + \frac{\kappa_i}{\varphi}\eta_i) + (1+\kappa_i)\frac{s_i + \frac{\kappa_i}{\varphi}\eta_i}{\frac{\kappa_{i+1}}{\varphi}}}{2(1+\kappa_i)} + \frac{\eta_{i+1}}{2} = \varphi\frac{1 + \frac{1+\kappa_i}{\kappa_{i+1}}}{2(1+\kappa_i)}(s_i + \frac{\kappa_i}{\varphi}\eta_i) + \frac{\eta_{i+1}}{2}$$

$$= \varphi\frac{\kappa_i(1+\kappa_i) + 1 + \kappa_i}{2\kappa_i(1+\kappa_i)^2}(s_i + \frac{\kappa_i}{\varphi}\eta_i) + \frac{\eta_{i+1}}{2} = \frac{\varphi}{2\kappa_i}(s_i + \frac{\kappa_i}{\varphi}\eta_i) + \frac{\eta_{i+1}}{2}$$

$$= \frac{\varphi}{2\kappa_i}\frac{q^i_{i,i+1,\tau} - b^i_{i,i+1}p^i_{i,i+1,\tau}}{a_i} + \frac{\eta_{i+1}}{2}.$$

Then I show that for dealer i, given dealer $i+1$'s strategy, it does not have incentive to deviate. Given dealer $i+1$'s strategy, the only way dealer i can affect the price that the trade takes place, $\frac{\varphi}{2\kappa_i}\frac{q^i_{i,i+1,\tau} - b^i_{i,i+1}p^i_{i,i+1,\tau}}{a^i_{i,i+1}} + \frac{\eta_{i+1}}{2}$, is to change $\frac{q^i_{i,i+1,\tau} - b^i_{i,i+1}p^i_{i,i+1,\tau}}{a^i_{i,i+1}} := A$. Thus dealer i solves

$$\max_{q^i_{i,i+1,\tau+1}} \mathbb{E}[q^i_{i,i+1,\tau+1}(\theta + \eta_i - \frac{\varphi}{2\kappa_i}A - \frac{\eta_{i+1}}{2})|s_i, \eta_i],$$

s.t.

$$q^i_{i,i+1,\tau+1} + b^{i+1}_{i,i+1}(\frac{\varphi}{2\kappa_i}A + \frac{\eta_{i+1}}{2}) + c^{i+1}_{i,i+1}\eta_{i+1} + \beta(\frac{\varphi}{2\kappa_i}A + \frac{\eta_{i+1}}{2}) = 0,$$

thus dealer i solves

$$\max_A \mathbb{E}[-(b^{i+1}_{i,i+1}(\frac{\varphi}{2\kappa_i}A + \frac{\eta_{i+1}}{2}) + c^{i+1}_{i,i+1}\eta_{i+1} + \beta(\frac{\varphi}{2\kappa_i}A + \frac{\eta_{i+1}}{2}))(\theta + \eta_i - \frac{\varphi}{2\kappa_i}A - \frac{\eta_{i+1}}{2})|s_i, \eta_i],$$

by construction,

$$b^{i+1}_{i,i+1} = \beta(\kappa_i - 1),$$

$$c^{i+1}_{i,i+1} = -\beta\kappa_i,$$

thus

$$\mathbb{E}[-(b^{i+1}_{i,i+1}(\frac{\varphi}{2\kappa_i}A + \frac{\eta_{i+1}}{2}) + c^{i+1}_{i,i+1}\eta_{i+1} + \beta(\frac{\varphi}{2\kappa_i}A + \frac{\eta_{i+1}}{2}))(\theta + \eta_i - \frac{\varphi}{2\kappa_i}A - \frac{\eta_{i+1}}{2})|s_i, \eta_i]$$

$$= \mathbb{E}[-(\beta(\kappa_i - 1)(\frac{\varphi}{2\kappa_i}A + \frac{\eta_{i+1}}{2}) - \beta\kappa_i\eta_{i+1} + \beta(\frac{\varphi}{2\kappa_i}A + \frac{\eta_{i+1}}{2}))(\theta + \eta_i - \frac{\varphi}{2\kappa_i}A - \frac{\eta_{i+1}}{2})|s_i, \eta_i]$$

$$= -\mathbb{E}[(\beta\kappa_i(\frac{\varphi}{2\kappa_i}A + \frac{\eta_{i+1}}{2}) - \beta\kappa_i\eta_{i+1})(\theta + \eta_i - \frac{\varphi}{2\kappa_i}A - \frac{\eta_{i+1}}{2})|s_i, \eta_i]$$

$$= -\mathbb{E}[(\beta(\frac{\varphi}{2}A - \frac{\kappa_i}{2}\eta_{i+1})(\theta + \eta_i - \frac{\varphi}{2\kappa_i}A - \frac{\eta_{i+1}}{2})|s_i, \eta_i]$$

$$= -\beta\kappa_i\mathbb{E}[-\frac{\varphi^2}{4\kappa_i^2}A^2 + \frac{\varphi}{2\kappa_i}A(\theta + \eta_i) - \frac{\eta_{i+1}}{2}(\theta + \eta_i - \frac{\eta_{i+1}}{2})|s_i, \eta_i],$$

thus

$$A = \frac{\kappa_i}{\varphi}(\mathbb{E}(\theta|s_i, \eta_i) + \eta_i),$$

thus

$$\frac{q^i_{i,i+1,\tau} - b^i_{i,i+1} p^i_{i,i+1,\tau}}{a^i_{i,i+1}} = \frac{\kappa_i}{\varphi}(\mathbb{E}(\theta|s_i, \eta_i) + \eta_i),$$

we have

$$q^i_{i,i+1,\tau} = a^i_{i,i+1}\frac{\kappa_i}{\varphi}(\mathbb{E}(\theta|s_i,\eta_i) + \eta_i) + b^i_{i,i+1}p^i_{i,i+1,\tau} = a^i_{i,i+1}\frac{\kappa_i}{\varphi}(\frac{\varphi}{\kappa_i}s_i + \eta_i) + b^i_{i,i+1}p^i_{i,i+1,\tau}$$

$$= a^i_{i,i+1}(s_i + \frac{\kappa_i}{\varphi}\eta_i) + b^i_{i,i+1}p^i_{i,i+1,\tau} = a^i_{i,i+1}s_i + b^i_{i,i+1}p^i_{i,i+1,\tau} - \beta\kappa_i\eta_i = a^i_{i,i+1}s_i + b^i_{i,i+1}p^i_{i,i+1,\tau} + c^i_{i,i+1}\eta_i.$$

■

Proof of Proposition 8

Proof.

Dealer i's trading strategy is $Q^i_{i,i+1} = a^i_{i,i+1}\hat{s}_i + b^i_{i,i+1}p + c^i_{i,i+1}\eta_i + d^i_{i,i+1}S$, dealer $i+1$'s trading strategy is $Q^{i+1}_{i,i+1} = b^{i+1}_{i,i+1}p + c^{i+1}_{i,i+1}\eta_{i+1} + d^{i+1}_{i,i+1}S$. Clients' demand is $\beta_{i,i+1}p + \delta_{i,i+1}S$.

For dealer i, market clearing implies

$$Q^i_{i,i+1} + b^{i+1}_{i,i+1}p + c^{i+1}_{i,i+1}\eta_{i+1} + d^{i+1}_{i,i+1}S + \beta_{i,i+1}p + \delta_{i,i+1}S = 0,$$

thus

$$p = -\frac{c^{i+1}_{i,i+1}\eta_{i+1} + d^{i+1}_{i,i+1}S + \delta_{i,i+1}S}{b^{i+1}_{i,i+1} + \beta_{i,i+1}} - \frac{Q^i_{i,i+1}}{b^{i+1}_{i,i+1} + \beta_{i,i+1}} := I^i_{i,i+1} + \lambda^i_{i,i+1}Q^i_{i,i+1}.$$

Dealer i's optimization problem is

$$\max_{Q^i_{i,i+1}}(\mathbb{E}(\theta_i|\hat{s}_i, \eta_i, S) - p)Q^i_{i,i+1} = (\mathbb{E}(\theta_i|s_i,\eta_i,S) - I_i - \lambda^i_{i,i+1}Q^i_{i,i+1})Q^i_{i,i+1},$$

FOC

$$-2\lambda^i_{i,i+1}Q^i_{i,i+1} + \mathbb{E}(\theta_i|s_i,\eta_i,S) - I_i = -2\lambda^i_{i,i+1}Q^i_{i,i+1} + \mathbb{E}(\theta_i|s_i,\eta_i,S) - p + \lambda^i_{i,i+1}Q^i_{i,i+1}$$

$$= -\lambda^i_{i,i+1}Q^i_{i,i+1} + \mathbb{E}(\theta_i|\hat{s}_i,\eta_i,S) - p = 0,$$

thus

$$Q^i_{i,i+1} = \frac{\mathbb{E}(\theta_i|s_i,\eta_i,S) - p}{\lambda^i_{i,i+1}}.$$

Using projection theorem,

$$\begin{pmatrix}\theta\\ \hat{s}_i\\ S\end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix}0\\0\\0\end{pmatrix}, \begin{pmatrix}\sigma^2_\theta & \sigma^2_\theta & \sigma^2_\theta\\ \sigma^2_\theta & Var(\hat{s}_i) & Cov(\hat{s}_i,S)\\ \sigma^2_\theta & Cov(\hat{s}_i,S) & Var(S)\end{pmatrix}\right],$$

$$\mathbb{E}(\theta_i|\hat{s}_i,\eta_i) = \frac{\sigma_\theta^2(Var(S) - Cov(\hat{s}_i,S))}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i,S)^2}\hat{s}_i + \frac{\sigma_\theta^2(Var(\hat{s}_i) - Cov(\hat{s}_i,S))}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i,S)^2}S + \eta_i,$$

we have

$$\begin{aligned} Q_{i,i+1}^i = -(b_{i,i+1}^{i+1} + \beta_{i,i+1})(&\frac{\sigma_\theta^2(Var(S) - Cov(\hat{s}_i,S))}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i,S)^2}\hat{s}_i \\ &+\frac{\sigma_\theta^2(Var(\hat{s}_i) - Cov(\hat{s}_i,S))}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i,S)^2}S + \eta_i - p). \end{aligned}$$

For dealer $i+1$, market clearing condition implies

$$Q_{i,i+1}^{i+1} + a_{i,i+1}^i\hat{s}_i + b_{i,i+1}^i p + c_{i,i+1}^i\eta_i + d_{i,i+1}^i S + \beta_{i,i+1}p + \delta_{i,i+1}S = 0,$$

thus

$$\begin{aligned} p &= -\frac{a_{i,i+1}^i\hat{s}_i + c_{i,i+1}^i\eta_i}{b_{i,i+1}^i + \beta_{i,i+1}} - \frac{d_{i,i+1}^i S + \delta_{i,i+1}S}{b_{i,i+1}^i + \beta_{i,i+1}} - \frac{Q_{i,i+1}^{i+1}}{b_{i,i+1}^i + \beta_{i,i+1}} \\ &:= I_{i+1} - \frac{d_{i,i+1}^i S + \delta_{i,i+1}S}{b_{i,i+1}^i + \beta_{i,i+1}} + \lambda_{i,i+1}^{i+1}Q_{i,i+1}^{i+1}. \end{aligned}$$

Dealer $i+1$'s optimization problem is

$$\begin{aligned} \max_{Q_{i,i+1}^{i+1}}(\mathbb{E}(\theta_{i+1}|I_{i+1},\eta_{i+1}) - p)Q_{i,i+1}^{i+1} = (&\mathbb{E}(\theta_{i+1}|I_{i+1},\eta_{i+1}) - I_{i+1} \\ &+\frac{d_{i,i+1}^i S + \delta_{i,i+1}S}{b_{i,i+1}^i + \beta_{i,i+1}} - \lambda_{i,i+1}^{i+1}Q_{i,i+1}^{i+1})Q_{i,i+1}^{i+1}, \end{aligned}$$

FOC implies

$$-2\lambda_{i,i+1}^{i+1}Q_{i,i+1}^{i+1} + \mathbb{E}(\theta_{i+1}|I_{i+1},\eta_{i+1}) - I_{i+1} + \frac{d_{i,i+1}^i S + \delta_{i,i+1}S}{b_{i,i+1}^i + \beta_{i,i+1}} = 0.$$

Using projection theorem,

$$\begin{pmatrix} \theta \\ \hat{s}_{i+1} = I_{i+1}(-\frac{b_{i,i+1}^i + \beta_{i,i+1}}{a_{i,i+1}^i}) \\ S \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 & \sigma_\theta^2 \\ \sigma_\theta^2 & Var(\hat{s}_{i+1}) & Cov(\hat{s}_{i+1},S) \\ \sigma_\theta^2 & Cov(\hat{s}_{i+1},S) & Var(S) \end{pmatrix}\right],$$

$$\begin{aligned} \mathbb{E}(\theta_{i+1}|\hat{s}_{i+1},\eta_i) = &\frac{\sigma_\theta^2(Var(S) - Cov(\hat{s}_{i+1},S))}{Var(\hat{s}_{i+1})Var(S) - Cov(\hat{s}_{i+1},S)^2}I_{i+1}(-\frac{b_{i,i+1}^i + \beta_{i,i+1}}{a_{i,i+1}^i}) \\ &+\frac{\sigma_\theta^2(Var(\hat{s}_{i+1}) - Cov(\hat{s}_{i+1},S))}{Var(\hat{s}_{i+1})Var(S) - Cov(\hat{s}_{i+1},S)^2}S + \eta_{i+1} \\ &= k_1 I_{i+1} + k_2 S + \eta_{i+1}, \end{aligned}$$

thus

$$-2\lambda_{i,i+1}^{i+1}Q_{i,i+1}^{i+1} + k_1 I_{i+1} + k_2 S + \eta_{i+1} - I_{i+1} + \frac{d_{i,i+1}^i S + \delta_{i,i+1}S}{b_{i,i+1}^i + \beta_{i,i+1}} = 0,$$

$$-2\lambda_{i,i+1}^{i+1}Q_{i,i+1}^{i+1}+(k_1-1)(p-\lambda_{i,i+1}^{i+1}Q_{i,i+1}^{i+1}+\frac{d_{i,i+1}^{i}S+\delta_{i,i+1}S}{b_{i,i+1}^{i}+\beta_{i,i+1}})+k_2S+\frac{d_{i,i+1}^{i}S+\delta_{i,i+1}S}{b_{i,i+1}^{i}+\beta_{i,i+1}}+\eta_{i+1}=0,$$

$$Q_{i,i+1}^{i+1}(-\lambda_{i,i+1}^{i+1})(1+k_1)+(k_1-1)p+k_2S+k_1\frac{d_{i,i+1}^{i}S+\delta_{i,i+1}S}{b_{i,i+1}^{i}+\beta_{i,i+1}}+\eta_{i+1}=0,$$

we have

$$Q_{i,i+1}^{i+1}=-(b_{i,i+1}^{i}+\beta_{i,i+1})(\frac{k_1-1}{k_1+1}p+\frac{k_2S+k_1\frac{d_{i,i+1}^{i}S+\delta_{i,i+1}S}{b_{i,i+1}^{i}+\beta_{i,i+1}}+\eta_{i+1}}{k_1+1}),$$

thus using

$$b_{i,i+1}^{i}=b_{i,i+1}^{i+1}+\beta_{i,i+1},$$

$$b_{i,i+1}^{i+1}=-(b_{i,i+1}^{i}+\beta_{i,i+1})\frac{k_1-1}{k_1+1},$$

we have

$$b_{i,i+1}^{i}=b_{i,i+1}^{i+1}+\beta_{i,i+1}=\frac{\beta_{i,i+1}}{k_1},$$

$$c_{i,i+1}^{i}=c_{i,i+1}^{i+1}=-\frac{\beta_{i,i+1}}{k_1},$$

we also have

$$k_1=\frac{\sigma_\theta^2(Var(S)-Cov(\hat{s}_{i+1},S)}{Var(\hat{s}_{i+1})Var(S)-Cov(\hat{s}_{i+1},S)^2}(-\frac{b_{i,i+1}^{i}+\beta_{i,i+1}}{a_{i,i+1}^{i}})$$

$$=\frac{\sigma_\theta^2(Var(S)-Cov(\hat{s}_{i+1},S)}{Var(\hat{s}_{i+1})Var(S)-Cov(\hat{s}_{i+1},S)^2}(-\frac{b_{i,i+1}^{i+1}+2\beta_{i,i+1}}{-(b_{i,i+1}^{i+1}+\beta_{i,i+1})\frac{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}})$$

$$=\frac{b_{i,i+1}^{i+1}+2\beta_{i,i+1}}{b_{i,i+1}^{i+1}+\beta_{i,i+1}}\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_{i+1},S)^2}{Var(\hat{s}_{i+1})Var(S)-Cov(\hat{s}_{i+1},S)^2},$$

thus

$$\frac{1}{k_1}=\frac{b_{i,i+1}^{i+1}+\beta_{i,i+1}}{b_{i,i+1}^{i+1}+2\beta_{i,i+1}}\frac{Var(\hat{s}_{i+1})Var(S)-Cov(\hat{s}_{i+1},S)^2}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_{i+1},S)^2},$$

using

$$Var(\hat{s}_{i+1})=Var(\hat{s}_i)+(\frac{c_{i,i+1}^{i}}{a_{i,i+1}^{i}})^2\sigma_\eta^2,$$

$$\frac{c_{i,i+1}^{i}}{a_{i,i+1}^{i}}=\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))},$$

we have

$$\frac{1}{k_1}=\frac{b_{i,i+1}^{i+1}+\beta_{i,i+1}}{b_{i,i+1}^{i+1}+2\beta_{i,i+1}}(1+(\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))})^2\sigma_\eta^2\frac{Var(S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_{i+1},S)^2}),$$

we have

$$b_{i,i+1}^{i+1}+\beta_{i,i+1}=\frac{b_{i,i+1}^{i+1}+\beta_{i,i+1}}{b_{i,i+1}^{i+1}+2\beta_{i,i+1}}$$

$$(1+(\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))})^2\sigma_\eta^2\frac{Var(S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_{i+1},S)^2})\beta_{i,i+1}$$

$$b_{i,i+1}^{i+1}+2\beta_{i,i+1}=\beta_{i,i+1}(1+\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma_\theta^4(Var(S)-Cov(\hat{s}_i,S))^2}\sigma_\eta^2 Var(S))$$

$$b_{i,i+1}^{i+1}+\beta_{i,i+1}=\beta_{i,i+1}\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma_\theta^4(Var(S)-Cov(\hat{s}_i,S))^2}\sigma_\eta^2 Var(S),$$

using

$$Var(\theta|S)=\sigma_\theta^2-\frac{\sigma_\theta^4}{Var(S)},$$

$$Var(\theta|\hat{s}_i,S)=\sigma_\theta^2-\sigma_\theta^4\frac{Var(S)-2Cov(\hat{s}_i,S)+Var(\hat{s}_i)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2},$$

we have

$$Var(\theta|S)-Var(\theta|\hat{s}_i,S)=\sigma_\theta^4\frac{(Var(S)-Cov(\hat{s}_i,S))^2}{(Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2)Var(S)},$$

thus

$$b_{i,i+1}^{i+1}+\beta_{i,i+1}=\frac{1}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}\sigma_\eta^2\beta_{i,i+1},$$

thus

$$k_1=\frac{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}{\sigma_\eta^2}.$$

Finally, we can solve

$$d_{i,i+1}^{i}=-\frac{\beta_{i,i+1}}{k_1}\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma_\theta^2=-\beta_{i,i+1}\frac{\sigma_\eta^2}{\sigma_\theta^2}\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{(Var(S)-Cov(\hat{s}_i,S))^2}Var(S),$$

$$\begin{aligned}
d_{i,i+1}^{i+1}&=-\beta_{i,i+1}(1+\frac{1}{k_1})(\frac{k_1\frac{d_{i,i+1}^{i}+\lambda_{i,i+1}^{i}}{b_{i,i+1}^{i}+\beta_{i,i+1}}+k_2}{k_1+1})\\
&=-\beta_{i,i+1}(1+\frac{1}{k_1})(\frac{k_1\frac{-\frac{\beta_{i,i+1}}{k_1}\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma_\theta^2+\lambda_{i,i+1}^{i}}{\beta_{i,i+1}(\frac{1}{k_1}+1)}+k_2}{k_1+1})\\
&=-\frac{\beta_{i,i+1}}{k_1}(k_1\frac{-\frac{\beta_{i,i+1}}{k_1}\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma_\theta^2+\lambda_{i,i+1}^{i}}{\beta_{i,i+1}(\frac{1}{k_1}+1)}+k_2)\\
&=-\frac{\beta_{i,i+1}}{k_1}(\frac{-\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma_\theta^2+\frac{k_1}{\beta_{i,i+1}}\lambda_{i,i+1}^{i}}{(\frac{1}{k_1}+1)}+k_2)\\
&=\frac{\beta_{i,i+1}}{1+k_1}(\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma_\theta^2-\frac{k_1}{\beta_{i,i+1}}\lambda_{i,i+1}^{i})-\frac{k_2}{k_1}\beta_{i,i+1}\\
&=\frac{\beta_{i,i+1}}{1+k_1}(\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma_\theta^2-\frac{k_1}{\beta_{i,i+1}}\lambda_{i,i+1}^{i})
\end{aligned}$$

$$-\frac{\beta_{i,i+1}}{k_1}\frac{\sigma_\theta^2(Var(\hat{s}_{i+1})-Cov(\hat{s}_{i+1},S))}{Var(\hat{s}_{i+1})Var(S)-Cov(\hat{s}_{i+1},S)^2}$$

$$=\frac{\beta_{i,i+1}}{1+k_1}(\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma_\theta^2-\frac{k_1}{\beta_{i,i+1}}\lambda_{i,i+1}^i)$$

$$-\frac{\beta_{i,i+1}}{k_1}\frac{\sigma_\theta^2(Var(\hat{s}_i)+(\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))})^2\sigma_\eta^2-Cov(\hat{s}_i,S))}{Var(\hat{s}_i)Var(S)+(\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))})^2\sigma_\eta^2 Var(S)-Cov(\hat{s}_i,S)^2}$$

$$=\frac{\beta_{i,i+1}}{1+k_1}(\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma_\theta^2-\frac{k_1}{\beta_{i,i+1}}\lambda_{i,i+1}^i)$$

$$-\frac{\beta_{i,i+1}}{k_1}\frac{\sigma_\theta^2(Var(\hat{s}_i)+(\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))})^2\sigma_\eta^2-Cov(\hat{s}_i,S))}{(Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2)(1+(\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{(\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S)))^2}\sigma_\eta^2)}$$

$$=\frac{\beta_{i,i+1}}{1+k_1}(\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma_\theta^2-\frac{k_1}{\beta_{i,i+1}}\lambda_{i,i+1}^i)$$

$$-\frac{\beta_{i,i+1}}{k_1}\frac{\sigma_\theta^2(Var(\hat{s}_i)+(\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))})^2\sigma_\eta^2-Cov(\hat{s}_i,S))}{(Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2)(1+\frac{1}{k_1})}$$

$$=\frac{\beta_{i,i+1}}{1+k_1}\frac{1}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma_\theta^2\big(Var(\hat{s}_i)-Cov(\hat{s}_i,S)-Var(\hat{s}_i)$$

$$-(\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))})^2\sigma_\eta^2+Cov(\hat{s}_i,S)\big)-\frac{\beta_{i,i+1}}{1+k_1}\frac{k_1}{\beta_{i,i+1}}\lambda_{i,i+1}^i$$

$$=\frac{\beta_{i,i+1}}{1+k_1}\frac{1}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma_\theta^2\big(-(\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))})^2\sigma_\eta^2)\big)$$

$$-\frac{k_1}{1+k_1}\lambda_{i,i+1}^i$$

$$=-\frac{\beta_{i,i+1}}{1+k_1}\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{(Var(S)-Cov(\hat{s}_i,S))^2}\varphi-\frac{k_1}{1+k_1}\lambda_{i,i+1}^i$$

$$=-\frac{\beta_{i,i+1}}{1+k_1}\frac{\sigma_\theta^4}{(Var(\theta|S)-Var(\theta|\hat{s}_i,S))Var(S)}\varphi-\frac{k_1}{1+k_1}\lambda_{i,i+1}^i$$

$$=-\frac{\beta_{i,i+1}}{1+\frac{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}{\sigma_\eta^2}}\frac{\sigma_\theta^4}{(Var(\theta|S)-Var(\theta|\hat{s}_i,S))Var(S)}\varphi-\frac{\frac{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}{\sigma_\eta^2}}{1+\frac{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}{\sigma_\eta^2}}\lambda_{i,i+1}^i.$$

For client,

$$\max_q \mathbb{E}\big((\theta-p)q-\frac{\mu}{2}q^2|p,S\big),$$

thus

$$q=\frac{\mathbb{E}(\theta|p,S)-p}{\mu},$$

using the results above,

$$b_{i,i+1}^i=b_{i,i+1}^{i+1}+\beta_{i,i+1}=\frac{\beta_{i,i+1}}{k_1},$$

$$c^i_{i,i+1} = c^{i+1}_{i,i+1} = -\frac{\beta_{i,i+1}}{k_1},$$
$$\frac{c^i_{i,i+1}}{a^i_{i,i+1}} = \frac{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}{\sigma^2_\theta(Var(S) - Cov(\hat{s}_i, S))},$$
$$k_1 = \frac{Var(\theta|S) - Var(\theta|\hat{s}_i, S)}{\sigma^2_\eta},$$
$$d^i_{i,i+1} = -\beta_{i,i+1}\frac{\sigma^2_\eta}{\sigma^2_\theta}\frac{Var(\hat{s}_i) - Cov(\hat{s}_i, S)}{(Var(S) - Cov(\hat{s}_i, S))^2}Var(S),$$
$$d^{i+1}_{i,i+1} = -\frac{\beta_{i,i+1}}{1 + \frac{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}{\sigma^2_\eta}}\frac{\sigma^4_\theta}{(Var(\theta|S) - Var(\theta|\hat{s}_i, S))Var(S)}\varphi$$
$$-\frac{\frac{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}{\sigma^2_\eta}}{1 + \frac{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}{\sigma^2_\eta}}\lambda^i_{i,i+1},$$

we have

$$p = -\frac{a^i_{i,i+1}}{b^i_{i,i+1} + b^{i+1}_{i,i+1} + \beta_{i,i+1}}\hat{s}_i - \frac{c^i_{i,i+1}\eta_i + c^{i+1}_{i,i+1}\eta_{i+1}}{b^i_{i,i+1} + b^{i+1}_{i,i+1} + \beta_{i,i+1}} - \frac{d^i_{i,i+1} + d^{i+1}_{i,i+1} + \lambda^i_{i,i+1}}{b^i_{i,i+1} + b^{i+1}_{i,i+1} + \beta_{i,i+1}}S$$

$$= -\frac{c^i_{i,i+1}\frac{\sigma^2_\theta(Var(S)-Cov(\hat{s}_i,S))}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}}{2\frac{\beta_{i,i+1}}{k_1}}\hat{s}_i - \frac{-\frac{\beta_{i,i+1}}{k_1}}{2\frac{\beta_{i,i+1}}{k_1}}(\eta_i+\eta_{i+1}) - \Big(-\beta_{i,i+1}\frac{\sigma^2_\eta}{\sigma^2_\theta}\frac{Var(\hat{s}_i) - Cov(\hat{s}_i, S)}{(Var(S) - Cov(\hat{s}_i, S))^2}Var(S)$$
$$-\frac{\beta_{i,i+1}}{1 + \frac{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}{\sigma^2_\eta}}\frac{\sigma^4_\theta}{(Var(\theta|S) - Var(\theta|\hat{s}_i, S))Var(S)}\varphi$$
$$-\frac{\frac{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}{\sigma^2_\eta}}{1 + \frac{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}{\sigma^2_\eta}}\lambda^i_{i,i+1} + \lambda^i_{i,i+1}\Big)\frac{1}{2\frac{\beta_{i,i+1}}{k_1}}S$$
$$= \frac{\frac{\sigma^2_\theta(Var(S)-Cov(\hat{s}_i,S))}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}}{2}\hat{s}_i + \frac{1}{2}(\eta_i + \eta_{i+1}) - \Big(-\beta_{i,i+1}\frac{\sigma^2_\eta}{\sigma^2_\theta}\frac{Var(\hat{s}_i) - Cov(\hat{s}_i, S)}{(Var(S) - Cov(\hat{s}_i, S))^2}Var(S)$$
$$-\frac{\beta_{i,i+1}}{1 + \frac{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}{\sigma^2_\eta}}\frac{\sigma^4_\theta}{(Var(\theta|S) - Var(\theta|\hat{s}_i, S))Var(S)}\varphi$$
$$+\frac{1}{1 + \frac{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}{\sigma^2_\eta}}\lambda^i_{i,i+1}\Big)\frac{1}{2\frac{\beta_{i,i+1}}{k_1}}S$$
$$:= x\hat{s}_i + yS + \frac{\eta_i + \eta_{i+1}}{2}.$$

$$\begin{pmatrix} \theta \\ \frac{p-yS}{x} \\ S \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_\theta & \sigma^2_\theta & \sigma^2_\theta \\ \sigma^2_\theta & Var(\hat{s}_i) + \frac{1}{2x^2}\sigma^2_\eta & Cov(\hat{s}_i, S) \\ \sigma^2_\theta & Cov(\hat{s}_i, S) & Var(S) \end{pmatrix}\right],$$

thus

$$\mathbb{E}(\theta|p,S)=\sigma_\theta^2\frac{(Var(S)-Cov(\hat{s}_i,S))\frac{p-yS}{x}+(Var(\hat{s}_i)+\frac{1}{2x^2}\sigma_\eta^2-Cov(\hat{s}_i,S))S}{Var(S)(Var(\hat{s}_i)+\frac{1}{2x^2}\sigma_\eta^2)-Cov(\hat{s}_i,S)^2},$$

thus

$$\begin{aligned}
\beta_{i,i+1} &= (\sigma_\theta^2\frac{(Var(S)-Cov(\hat{s}_i,S))\frac{1}{x}}{Var(S)(Var(\hat{s}_i)+\frac{1}{2x^2}\sigma_\eta^2)-Cov(\hat{s}_i,S)^2}-1)\frac{1}{\mu}\\
&= (\sigma_\theta^2\frac{(Var(S)-Cov(\hat{s}_i,S))\frac{1}{\frac{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))}{\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{2}}}}{Var(S)(Var(\hat{s}_i)+\frac{1}{2(\frac{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))}{\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{2}})^2}\sigma_\eta^2)-Cov(\hat{s}_i,S)^2}-1)\frac{1}{\mu}\\
&= (\frac{2(Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2)}{Var(S)(Var(\hat{s}_i)+\frac{2}{(\frac{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2})^2}\sigma_\eta^2)-Cov(\hat{s}_i,S)^2}-1)\frac{1}{\mu}\\
&= (\frac{2(Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2)}{Var(S)Var(\hat{s}_i)-Cov(\hat{s}_i,S)^2+Var(S)\frac{2}{(\frac{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2})^2}\sigma_\eta^2}-1)\frac{1}{\mu}\\
&= (\frac{2}{1+Var(S)\frac{2(Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2)}{\sigma_\theta^4(Var(S)-Cov(\hat{s}_i,S))^2}\sigma_\eta^2}-1)\frac{1}{\mu}\\
&= (\frac{2}{1+2\frac{\sigma_\eta^2}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}}-1)\frac{1}{\mu}=(\frac{1-2\frac{\sigma_\eta^2}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}}{1+2\frac{\sigma_\eta^2}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}})\frac{1}{\mu},
\end{aligned}$$

$$\lambda_{i,i+1}^i=\sigma_\theta^2\frac{(Var(S)-Cov(\hat{s}_i,S))\frac{-y}{x}+(Var(\hat{s}_i)+\frac{1}{2x^2}\sigma_\eta^2-Cov(\hat{s}_i,S))}{Var(S)(Var(\hat{s}_i)+\frac{1}{2x^2}\sigma_\eta^2)-Cov(\hat{s}_i,S)^2}\frac{1}{\mu},$$

thus we have

$$\begin{aligned}
c_{i,i+1}^{i+1} &= (\frac{2}{1+2\frac{\sigma_\eta^2}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}}-1)\frac{1}{\mu}(-\frac{\sigma_\eta^2}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}),\\
&= (1-\frac{2}{1+2\frac{\sigma_\eta^2}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}})\frac{1}{\mu}\frac{\sigma_\eta^2}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}.
\end{aligned}$$

As the second order condition requires that

$$b_{i,i+1}^{i+1}+\beta_{i,i+1}=\frac{1}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}\sigma_\eta^2\beta_{i,i+1}$$

$$=\frac{1}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}\sigma_\eta^2(\frac{1-2\frac{\sigma_\eta^2}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}}{1+2\frac{\sigma_\eta^2}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}})\frac{1}{\mu}<0,$$

which requires $\frac{\sigma_\eta^2}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}>\frac{1}{2}$. It's immediately clear that $\beta_{i,i+1}$ is decreasing in $\frac{1}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}$ and the surplus from the asset reallocation $c_{i,i+1}^{i+1}\sigma_\eta^2$ in increasing in $\frac{1}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}$.

■

4.1.2 Microfoundation of Clients' Linear Demand

In the trade between dealer i and $i+1$, a continuum of clients participate in this trade. Clients do not have private information about the asset payoff or the dealers' private values of the asset. Clients are risk-neutral and have liquidity needs of the asset. Client j's value of the asset is η_j with $\mathbb{E}(\eta_j) = 0$ and independently distributed across the clients. Clients incur a quadratic flow cost of trading the asset.

The demand function of client j is $q^j_{i,i+1}(p_{i,i+1}, \eta_j)$.[1]

The payoff of client j is

$$\mathbb{E}\big(\eta_j q^j_{i,i+1} - p_{i,i+1} q^j_{i,i+1} - \frac{\mu}{2}(q^j_{i,i+1})^2\big).$$

Thus we have that the demand of client j is

$$q^j_{i,i+1}(p_{i,i+1}, \eta_j) = \frac{1}{\mu}(\eta_j - p_{i,i+1}).$$

Thus the aggregate demand of clients on each link $(i, i+1)$ is $\beta p_{i,i+1} \equiv -\frac{2}{\mu} p_{i,i-1}$.

It generates a linear demand function of clients, which ensures the existence of linear equilibrium for bilateral trade in a double auction. So this provides a micro-foundation for the reduced-form linear demand function of the customer base in Babus and Kondor (2018), Babus, Kondor and Wang (2019).

By assuming that clients' value of the asset does not depend on θ, I shut down clients' learning of the asset payoff from the trading price. In the extension section, I relax the baseline assumption that clients are liquidity traders and assume that clients trade also for the asset payoff. In that case, the information diffusion in the tree network and the equilibrium price function of each link is the same as the baseline model. The details are shown in the proof of Proposition 8. But as clients learn from the price, clients' trading intensity is affected by the information content in the price. As shown in Proposition 8, the main result in this paper still holds and is amplified in this case.

[1]The clients can submit their demand schedules via dealers i and $i+1$, who do dual-capacity trading by submitting clients' orders and trading on their own accounts. See Fishman and Longstaff (1992), Roell (1990).

4.1.3 Steps from the Academic TRACE File to the Cleaned Academic TRACE Sample

Eliminate trade between a dealer and customer

As we focus on inter-dealer trade, we drop the trade between a dealer and customer. There are 11,900,080 unique trade reports on 32,795 different CUSIPs left in the dataset.

Eliminate bonds based on characteristics

I replicate the cleaning procedure in Asquith, Covert and Pathak (2019). Before eliminating and correcting trade reports, we match the TRACE dataset to the universe of corporate bonds in the Mergent FISD database. The Mergent FISD database is our source for bond characteristics such as issue size, ratings, maturity, etc. which we add to the Academic TRACE dataset. The Mergent FISD database we use includes all the bonds with an offering date between January of 1950 and January of 2010.

I drop all TRACE bonds that do not match to FISD by CUSIP. There are 11,800,641 unique trade reports on 31,682 different CUSIPs left in the dataset.

I also drop all bonds with equity-like characteristics (convertibles, exchangeables, etc.) since their equity component may be included in the bond price. There are 11,005,869 unique trade reports on 29,872 different CUSIPs left in the dataset.

I next drop all Rule 144a bonds because TRACE did not disseminate trading information on these bonds during 2002-2005.[2] There are 10,704,976 unique trade reports on 26,965 different CUSIPS left.

FISD does not report a correct issue size in some cases. For example, there are some bonds in FISD with a reported issue size of 0. I drop all bonds with a reported FISD issue size of less than 1. (offering-amt $\leq$1) There are 10,687,352 unique trade reports on 26,729 different CUSIPs left.

Eliminate trade reports because of self-reported errors

Next, we eliminate trade reports which do not take place as reported since they are later modified, cancelled, or reversed. I replicate cleaning procedure steps 1 and 2 in

[2] One way to circumvent TRACE, which applies to publicly issued bonds, is for a firm to issue privately placed bonds (sometimes referred to as Rule 144a securities, for the section of the Securities Act of 1933 that provides exemption from registration requirements).

Dick-Nielsen (2014).

I first clean same-day corrections and cancellations. Same-day refers to corrections and cancelations reported within the same reporting date (not transaction date). These can be uniquely identified by the link between the Record Count Number and Original Message Sequence Number. The Record Count Number is unique on an intra-reporting day level. There are 10,423,151 unique trade reports on 26,647 different CUSIPs left.

Remove reversals and the matching original transaction report. Reversals are cancelations reported on a later date than the date on which the orignal transaction took place. Trade reversals are identified by the As Of Indicator field by "R". Since the original trade and its reversal are reported to TRACE on different days, the Original Message Sequence Number of reversal trades do not necessarily match the Record Count Number of the original trade. Therefore, to link a reversal to its original trade, following Asquith, Covert and Pathak (2019), we match the reports using eight identifying characteristics: CUSIP, Trade Execution Date, Trade Execution Time, Reported Price, Entered Volume Quantity, Reporter ID, Contraparty ID, Buy/Sell Indicator, Buyer Capacity, and Seller Capacity. (Nick-Nielsen (2014) does not use Reporter ID, Buyer Capacity, Seller Capacity.) I called matches with these criteria a "ten-way" match.

In the dataset of reversals, these ten characteristics identify a unique observation for most of the observations. For those that are not uniquely identified, we keep the last one by the reported date and time. I keep those with a trade reported date after the trade execution date. Then we merge the dataset of original trades using the dataset of reversals. If there is only one exact match, both the reversal and its matched trade are eliminated. If there is more than one exact match, we eliminate the reversal trade and one of the matching trades. Because these multiple matching trades occur at the same time, date, price and volume, the cleaned dataset is unaffected by the choice of which matching trade reports we eliminate. There are 10,121,712 unique trade reports on 26,581 different CUSIPs left.

Not all reversals have an exact ten-way match. I then drop the same execution time requirement. Since execution time is self-reported, we assume these nine-way matches were the original trades that were meant to be reversed, and we eliminate the reversal and the matched trade following the steps above. Still, before we merge the datasets, in the dataset of reversals left, for those that are not uniquely identified by those nine characteristics, we keep the last one by the reported date and time. There are 10,113,094 unique trade reports on 26,578 different CUSIPs left.

Next, we relax the requirement that the price must be exact in a nine-way reversal and look for matches when prices are rounded to 0.01. There are 10,112,431 unique trade reports on 26,578 different CUSIPs left.

There are 16859 reversals (each of them is uniquely identified by CUSIP, Trade Execution Date, Reported Price (rounded to 0.01), Entered Volume Quantity, Reporter ID, Contraparty ID, Buy/Sell Indicator, Buyer Capacity, and Seller Capacity) that we are unable to match to an original trade. I dropped these reversal reports from the dataset.

Eliminate one of the sides for inter-dealer trade

There are multiple conventions or paths by which the transacting parties can be reported to TRACE. In addition to self-reporting, TRACE also allows another party (such as a clearing firm) to fulfill the reporting obligation of the transacting party. There are three fields that are used when trade reports do not report the transacting dealers in the trade. They are Reporter Give-up ID, Contraparty Give-up ID, and Locked-in Trade Identifier. In a Give-up trade, a clearing firm can submit a report on behalf of either of the transacting dealers. When this is the case, either the Reporter Give-up ID or Contraparty Give-up ID is populated by the ID of the transacting dealer. To correctly identify the transacting parties in these trades, we replace the Reporter ID (Contraparty ID) with the Reporter Give-up ID (Contraparty Give-up ID).

The other case where TRACE allows a variance on its reporting requirements is a locked-in trade report. In a locked-in trade report, the reporting party submits the trade report on its own behalf as well as on behalf of the contraparty. That is, there is only one trade report, rather than a separate report from the buying dealer and selling dealer. When the Locked-in Trade Identifier is checked, the reporter ID and the contraparty ID are the same, but one give-up field is populated. For each trade report where the Locked-in Flag is marked, we follow the convention in the paragraph above for the give-up fields and modify the Reporter or Contraparty ID fields in the existing trade report.

For regular trade (Regular Trade Identifier is "x"), we adopt the convention of preserving the sell-side report. For locked-in trade, the trade report is kept. There are 5,155,102 unique trade reports on 25,239 different CUSIPs left.

Address trade splitting

I basically replicate the procedure in Asquith, Covert and Pathak (2019). Dealers might report one trade in multiple pieces. To deal with trade splitting, we aggregate all of the reports with the same CUSIP, Execution Date, Price, Reported ID, Contraparty ID, Buy/Sell Indicator, Buyer Capacity, and Seller Capacity. There are 4,889,149 unique trade reports and 25,239 different CUSIPs left.

Eliminate trade reports with price or volume issues

I basically replicate the procedure in Asquith, Covert and Pathak (2019). Some trade prices are vastly out of line with other prices for the bond during the same period. The reference prices are the median prices of the same bond traded in the same month. A trade price is vastly out of line if the bond price differs by more than $20 per bond. I eliminate the reports with price vastly out of line. There are 4,885,676 unique trade reports and 25,239 different CUSIPs left.

The procedure above does not eliminate the report if there is not another trade report in that month. Therefore we drop trade reports less than the 0.01 percentile and greater than 99.99 percentile of all trade prices in the sample. Next, we eliminate trades when volume is less than the 0.01 percentile and greater than 99.99 percentile of all trade volumes in the sample. There are 4,883,688 unique trade reports and 25,181 different CUSIPs left.

Eliminate if the trade is under special circumstances, or the traded asset is an equity linked note, or the trade is not a cash sale

I replicate Dick-Nielsen (2014).There are 4,865,045 unique trade reports and 25,101 different CUSIPs left.

4.2 Appendix of Chapter 2

4.2.1 Proofs of Propositions and Lemmas

Proof of Proposition 9

Proof.

Define $\sigma_p := \frac{\gamma\sigma_\varepsilon}{\omega\alpha_x}$, $\alpha_p := \sigma_p^{-2}$ denotes the precision of the public signal p, $\alpha := \alpha_x + \alpha_p$.

We guess in equilibrium,

$$p = \theta - \frac{\gamma\sigma_\varepsilon}{\omega\alpha_x}\varepsilon + \frac{z(p)\gamma}{\omega\alpha}.$$

Small traders hold belief $z^e(p)$ of the large trader's demand schedule.

$$\tilde{p} := p - \frac{\gamma z^e(p)}{\omega\alpha}.$$

Small trader i's asset demand is,

$$k(x_i, p) = \frac{E[\theta|x_i, p, z^e(p)] - p}{\gamma Var[\theta|x_i, p, z^e(p)]} = \frac{\alpha_x x_i + \alpha_p \tilde{p} - \alpha p}{\gamma}.$$

The aggregate demand of small traders in the asset market is,

$$K(\theta, p) = E(k(x_i, p)|\theta, p)\omega = \frac{\alpha_x\theta + \alpha_p\tilde{p} - \alpha p}{\gamma}\omega.$$

Market clear condition,

$$K(\theta, p) + z(p) = \sigma_\varepsilon\varepsilon.$$

It implies

$$z(p) = M + Dp - B\tilde{p},$$

where $M := (\theta - \frac{\gamma\sigma_\varepsilon}{\omega\alpha_x}\varepsilon)(-\frac{\omega\alpha_x}{\gamma})$, $D := \frac{\omega\alpha}{\gamma}$, $B := \frac{\omega\alpha_p}{\gamma}$. Notice if substituting $\tilde{p}$ with $p - \frac{\gamma z^e(p)}{\omega\alpha}$,

$$p = \theta - \frac{\gamma\sigma_\varepsilon}{\omega\alpha_x}\varepsilon + \frac{\gamma}{\omega\alpha_x}(z(p) - \frac{\alpha_p}{\alpha}z^e(p)),$$

thus if $z(p) = z^e(p)$, our initial guess is verified.

Conditional on M, the larger trader's expected payoff in the asset market,

$$U_1(M, \tilde{p}) = (E[\theta|M] - p)z(p) = \eta(\frac{M}{B-D} - p(\tilde{p}))(M + Dp(\tilde{p}) - B\tilde{p}).$$

In equilibrium,

$$\tilde{p}(M) = \frac{M}{B-D} = \theta - \frac{\gamma\sigma_\varepsilon}{\omega\alpha_x}\varepsilon$$

$$\Rightarrow \frac{dU_1(M,\tilde{p})}{d\tilde{p}}|_{\tilde{p}=\frac{M}{B-D}} = 0, \quad \frac{d^2U_1(M,\tilde{p})}{d\tilde{p}^2}|_{\tilde{p}=\frac{M}{B-D}} \leq 0$$

$$\Rightarrow p(\tilde{p}) = \tilde{p}.$$

Next we check the sufficient condition. If $p(\tilde{p}) = \tilde{p}$, $\frac{d^2 U_1(M,\tilde{p})}{d\tilde{p}^2} \leq 0 \quad \forall M, \tilde{p}$.

It's quite intuitive to have this result immediately after seeing the price function $p = \theta - \frac{\gamma\sigma_\varepsilon}{\omega\alpha_x}\varepsilon + \frac{z(p)\gamma}{\omega\alpha}$. For the large trader, conditional on any exogenous M, any purchase of assets will raise the price to be higher than the expected value of dividend $E(\theta|M)$, thus the large trader won't purchase; and any sale of assets will depress the price to be lower than the expected value of dividend, thus the large trader won't sell. Or in other words, any nonzero position for the first-stage large trader in asset market will affect the asset price in his disadvantage. Thus not participating in the asset market at all is the optimal strategy for the first-stage large trader.

Thus in equilibrium,

$$p = \theta - \frac{\gamma\sigma_\varepsilon}{\omega\alpha_x}\varepsilon.$$

■

Proof of Proposition 10

Proof.

Under Assumption 1,

$$\frac{d\bar{\theta}(p)}{dp} = \frac{\alpha_p}{\alpha_p - \frac{\sqrt{\alpha}_x}{1-\lambda}\frac{1}{\phi(\Phi^{-1}(\frac{\bar{\theta}(p)-\lambda}{1-\lambda}))}} < 0.$$

$$\frac{d\hat{\theta}(p)}{dp} = \frac{\alpha_p}{\alpha_p - \frac{\sqrt{\alpha}_x}{(1-\lambda)}\frac{1}{\phi(\Phi^{-1}(\frac{\underline{\theta}(p)}{1-\lambda}))}}\frac{1 + \frac{1}{\sqrt{\alpha}_x(1-\lambda)\phi(\Phi^{-1}(\frac{\underline{\theta}(p)}{1-\lambda}))}}{1 + \frac{1}{\sqrt{\alpha}_x(1-\lambda)\phi(\Phi^{-1}(\frac{\hat{\theta}(p)}{1-\lambda}))}} < 0.$$

Thus $\Phi(\frac{\bar{\theta}(p)-p}{\sigma_p})$ and $\Phi(\frac{\hat{\theta}(p)-p}{\sigma_p})$ are decreasing in p.

From equations

$$(1-\lambda)\Phi(\frac{\underline{x}^*(p) - \underline{\theta}(p)}{\sigma_x}) = \underline{\theta}(p),$$

$$\lambda + (1-\lambda)\Phi(\frac{\underline{x}^*(p) - \hat{\theta}(p)}{\sigma_x}) = \hat{\theta}(p),$$

it's clear that for any $\underline{x}^*(p)$, $\hat{\theta} > \underline{\theta}$.

Combine equations

$$\lambda + (1-\lambda)\Phi(\frac{\underline{x}^*(p) - \hat{\theta}(p)}{\sigma_x}) = \hat{\theta}(p),$$

$$\Phi(\sqrt{\alpha_x+\alpha_p}(\underline{\theta}(p) - \frac{\alpha_x}{\alpha}\underline{x}^*(p) - \frac{\alpha_p}{\alpha}p)) = t,$$

we have

$$\frac{\alpha_p}{\alpha}\hat{\theta}(p) - \frac{\sqrt{\alpha}_x}{\alpha}\Phi^{-1}(\frac{\hat{\theta}(p)-\lambda}{1-\lambda}) - (\hat{\theta}(p) - \underline{\theta}(p)) = \frac{\Phi^{-1}(t)}{\sqrt{\alpha}} + \frac{\alpha_p}{\alpha}p.$$

Combine equations

$$\lambda + (1-\lambda)\Phi(\frac{\bar{x}^*(p) - \bar{\theta}(p)}{\sigma_x}) = \bar{\theta}(p),$$

$$\Phi(\sqrt{\alpha_x+\alpha_p}(\bar{\theta}(p) - \frac{\alpha_x}{\alpha}\bar{x}^*(p) - \frac{\alpha_p}{\alpha}p)) = t,$$

we have

$$\frac{\alpha_p}{\alpha}\bar{\theta}(p) - \frac{\sqrt{\alpha}_x}{\alpha}\Phi^{-1}(\frac{\bar{\theta}(p)-\lambda}{1-\lambda}) = \frac{\Phi^{-1}(t)}{\sqrt{\alpha}} + \frac{\alpha_p}{\alpha}p.$$

Under Assumption 1, LHS of equation above is decreasing in $\bar{\theta}$, thus $\bar{\theta}(p) > \hat{\theta}(p), \forall p$, thus $\Phi(\frac{\bar{\theta}(p)-p}{\sigma_p}) > \Phi(\frac{\hat{\theta}(p)-p}{\sigma_p}), \forall p$. Combined with the monotonicity of $\Phi(\frac{\bar{\theta}(p)-p}{\sigma_p})$ and $\Phi(\frac{\hat{\theta}(p)-p}{\sigma_p})$, we have $p_1 > p_2$.

■

Proof of Proposition 12

Lemma 9. *Given $p(\tilde{p})$ pinned down by (2.10) and some $(\tilde{p}^*, p_0)$, a sufficient condition for (9) to hold is $\frac{\partial^2 U(M,\tilde{p})}{\partial \tilde{p}\partial M} \leq 0 \ \forall \tilde{p} \leq \tilde{p}^*, \forall M \geq (B-D)\tilde{p}^*$.*[3]

Proof.

From $\frac{\partial^2 U(M,\tilde{p})}{\partial \tilde{p}\partial M} \leq 0$, and $\frac{\partial U(M,\tilde{p})}{\partial \tilde{p}}|_{\tilde{p}=\frac{M}{B-D}} = 0$, we have for any $M, M' \geq (B-D)\tilde{p}^*$,

$$\frac{\partial U(M',\tilde{p})}{\partial \tilde{p}}|_{\tilde{p}=\frac{M}{B-D}} \leq 0, \text{ if } M' > M; \frac{\partial U(M',\tilde{p})}{\partial \tilde{p}}|_{\tilde{p}=\frac{M}{B-D}} \geq 0, \text{ if } M' < M, \tag{4.2.1}$$

Thus, for any $M, M' \geq (B-D)\tilde{p}^*$, $U(M',\tilde{p})|_{\tilde{p}=\frac{M'}{B-D}} \geq U(M',\tilde{p})|_{\tilde{p}=\frac{M}{B-D}}$.

Otherwise, WLOG, assume $M < M'$, $\exists \tilde{p} \in [\frac{M'}{B-D}, \frac{M}{B-D}]$, such that $\frac{\partial U(M',\tilde{p})}{\partial \tilde{p}}|_{\tilde{p}\in[\frac{M'}{B-D},\frac{M}{B-D}]} > 0$, contradicting (4.2.1).

■

[3]This ensures the single-crossing property (Spence-Mirrlees Property) holds for $U(M, \tilde{p})$.

Lemma 10. *A sufficient condition for* $\frac{\partial^2 U(M,\tilde{p})}{\partial \tilde{p} \partial M} \leq 0 \ \forall \tilde{p} \leq \tilde{p}^*, M \geq (B-D)\tilde{p}^*$ *to hold is,* $p(\tilde{p})$ *pinned down above satisfies*

$$p(\tilde{p}) - \tilde{p} \geq \frac{(2D-B)\phi(\frac{\bar{\theta}(\tilde{p})-\tilde{p}}{\sigma_p})\frac{d\bar{\theta}(\tilde{p})}{d\tilde{p}}\frac{1}{\sigma_p}}{B^2\eta - \phi(1)\frac{d\bar{\theta}(\tilde{p})}{d\tilde{p}}\frac{1}{\sigma_p^2}2D}, \quad \forall \tilde{p} \leq \tilde{p}^*,$$

$p(\tilde{p})_{suf}$ *is defined by letting the above condition be binding.*

Proof.

$\frac{\partial^2 U(M,\tilde{p})}{\partial \tilde{p} \partial M} = \frac{B^2}{2D(D-B)}\eta + \frac{2D-B}{B-D}\frac{\phi(\frac{\bar{\theta}(\tilde{p})-\tilde{p}}{\sigma_p})\frac{d\bar{\theta}(\tilde{p})}{d\tilde{p}}\frac{1}{\sigma_p}}{2D(p(\tilde{p})-\tilde{p})} + \phi(\frac{\bar{\theta}(\tilde{p})-\frac{M}{B-D}}{\sigma_p})\frac{\bar{\theta}(\tilde{p})-\frac{M}{B-D}}{\sigma_p}\frac{d\bar{\theta}(\tilde{p})}{d\tilde{p}}\frac{1}{(B-D)\sigma_p^2}$.

Given $\frac{d\bar{\theta}(\tilde{p})}{d\tilde{p}} < 0$, $B < D$, and $\phi(\frac{\bar{\theta}(\tilde{p})-\frac{M}{B-D}}{\sigma_p})\frac{\bar{\theta}(\tilde{p})-\frac{M}{B-D}}{\sigma_p} \in [-\phi(1), \phi(1)]$, $\forall \tilde{p} \leq \tilde{p}^*, M \geq (B-D)\tilde{p}^*$, a sufficient condition for the condition in Lemma 9 to hold can be characterized.

■

Assumption 2. $\{\alpha_x, \omega, \gamma, \lambda, \eta, \varepsilon\}$ *satisfies*

$$\phi(1)(\frac{\alpha_x(1-\lambda)}{\alpha_x(1-\lambda)-\kappa\sqrt{2\pi\alpha_x}} - 1)\frac{\alpha_x(1-\lambda)}{\alpha_x(1-\lambda)-\kappa\sqrt{2\pi\alpha_x}} - \frac{1}{\sqrt{2\pi}}\frac{\kappa^{\frac{3}{2}}\alpha_x^2\phi(1)(1-\lambda)}{(\frac{\alpha_x(1-\lambda)}{\sqrt{2\pi}} - \kappa\sqrt{\alpha_x})^3}$$

$$\leq (\frac{1}{4(1+\kappa)} - \frac{1}{2})^2(1+\kappa)2\eta\frac{\omega}{\gamma},$$

$$1-\lambda \leq \frac{(2\kappa^2+2\kappa-\frac{1}{2})\kappa\eta\frac{\omega}{\gamma}}{\phi(1)2(1+\kappa)+(2\kappa^2+2\kappa-\frac{1}{2})\eta\frac{\omega}{\gamma}}\sqrt{\frac{2\pi}{\alpha_x}},$$

where $\kappa := \frac{\alpha_x}{\alpha_p} = \frac{\gamma^2\sigma_\varepsilon^2}{\omega^2\alpha_x}$.

For any $\alpha_x, \omega, \gamma, \lambda, \eta$, when κ is relatively large, Assumption 2 is satisfied. That implies when σ_ε is relatively large, Assumption 2 is satisfied.

Under Assumption 2, Lemma 11 shows the existence of $(\tilde{p}^*, p_0)$, $\tilde{p}^* \geq p_0$, such that the condition in Lemma 10 is satisfied.

Lemma 11. *Under Assumption 2, for any* $\tilde{p}^*$, *there exists* $(\tilde{p}^*, p_0)$, $\tilde{p}^* \geq p_0$, *such that* $p(\tilde{p})$ *pinned down by (2.10) and* $(\tilde{p}^*, p_0)$ *satisfies the condition in Lemma 10. The set of such* $(\tilde{p}^*, p_0)$ *is given by,*

$$\frac{\phi(\frac{\bar{\theta}(\tilde{p}^*)-\tilde{p}^*}{\sigma p})\frac{d\bar{\theta}(\tilde{p})}{d\tilde{p}}|_{\tilde{p}=\tilde{p}^*}}{(\frac{1}{2}-\frac{B}{4D})2\eta D\sigma_p} \leq p_0 - \tilde{p}^* \leq 0.$$

Proof.

For some constant $C > \frac{B}{2D}$, we can define an auxiliary locus $p(\tilde{p})_{aux}$ by

$$C = \frac{(p-\tilde{p})B\eta + \phi(\frac{\bar{\theta}(\tilde{p})-\tilde{p}}{\sigma p})\frac{d\bar{\theta}(\tilde{p})}{d\tilde{p}}\frac{1}{\sigma_p}}{(p(\tilde{p})_{aux}-\tilde{p})2\eta D},$$

we have $p(\tilde{p})_{aux} - \tilde{p} = \frac{\phi(\frac{\bar{\theta}(\tilde{p})-\tilde{p}}{\sigma p})\frac{d\bar{\theta}(\tilde{p})}{d\tilde{p}}}{(C-\frac{B}{2D})2\eta D\sigma_p}$, and $\frac{dp}{d\tilde{p}}|_{p(\tilde{p})_{aux}} = \frac{d(\phi(\frac{\bar{\theta}(\tilde{p})-\tilde{p}}{\sigma p})\frac{d\bar{\theta}(\tilde{p})}{d\tilde{p}})}{d\tilde{p}}\frac{1}{(C-\frac{B}{2D})2\eta D\sigma_p} + 1$.

Suppose C satisfies $\frac{dp}{d\tilde{p}}|_{p(\tilde{p})_{aux}} \geq C\ \forall \tilde{p} \leq \tilde{p}^*$, then $p(\tilde{p})_{aux}$ is the lower bound for any $p(\tilde{p})$ pinned down by (2.10) and $(\tilde{p}^*, p_0)$, where

$$\frac{\phi(\frac{\bar{\theta}(\tilde{p}^*)-\tilde{p}^*}{\sigma p})\frac{d\bar{\theta}(\tilde{p})}{d\tilde{p}}|_{\tilde{p}=\tilde{p}^*}}{(C-\frac{B}{2D})2\eta D\sigma_p} \leq p_0 - \tilde{p}^* \leq 0.$$

$\frac{dp}{d\tilde{p}}|_{p(\tilde{p})_{aux}} \geq C\ \forall \tilde{p} \leq \tilde{p}^*$ is equivalent to

$$\frac{d(\phi(\frac{\bar{\theta}(\tilde{p})-\tilde{p}}{\sigma p})\frac{d\bar{\theta}(\tilde{p})}{d\tilde{p}})}{d\tilde{p}} \geq (C-1)(C-\frac{B}{2D})2\eta D\sigma_p, \quad \forall \tilde{p} \leq \tilde{p}^*, \tag{4.2.2}$$

Let $C = \frac{1+\frac{B}{2D}}{2}$, RHS of (4.2.2) is minimized. Denote $\Phi^{-1}(\frac{\bar{\theta}(\tilde{p})-\lambda}{1-\lambda})$ by x.

$$\frac{d(\phi(\frac{\bar{\theta}(\tilde{p})-\tilde{p}}{\sigma p})\frac{d\bar{\theta}(\tilde{p})}{d\tilde{p}})}{d\tilde{p}} = -\phi(\frac{\bar{\theta}(\tilde{p})-\tilde{p}}{\sigma_p})\frac{\bar{\theta}(\tilde{p})-\tilde{p}}{\sigma_p}\frac{1}{\sigma_p}(\frac{\alpha_p(1-\lambda)\phi(x)}{\alpha_p(1-\lambda)\phi(x)-\sqrt{\alpha}_x}-1),$$

$$\frac{\alpha_p(1-\lambda)\phi(x)}{\alpha_p(1-\lambda)\phi(x)-\sqrt{\alpha}_x} + \phi(\frac{\bar{\theta}(\tilde{p})-\tilde{p}}{\sigma_p})\alpha_p^2\sqrt{\alpha}_x\frac{(1-\lambda)\phi(x)x}{(\alpha_p(1-\lambda)\phi(x)-\sqrt{\alpha}_x)^3}. \tag{4.2.3}$$

$\phi(\frac{\bar{\theta}(\tilde{p})-\tilde{p}}{\sigma_p})\frac{\bar{\theta}(\tilde{p})-\tilde{p}}{\sigma_p}, \phi(x)x \in [-\phi(1), \phi(1)]$. The lower bound of RHS of (4.2.3) is derived and we require it to be bounded by $(C-1)(C-\frac{B}{2D})2\eta D\sigma_p|_{C=\frac{1+\frac{B}{2D}}{2}}$, i.e.,

$$-(\frac{B}{4D}-\frac{1}{2})^2 2\eta D\sigma_p \leq -\phi(1)\frac{1}{\sigma_p}(\frac{\alpha_p(1-\lambda)\frac{1}{\sqrt{2\pi}}}{\alpha_p(1-\lambda)\frac{1}{\sqrt{2\pi}}-\sqrt{\alpha}_x}-1)\frac{\alpha_p(1-\lambda)\frac{1}{\sqrt{2\pi}}}{\alpha_p(1-\lambda)\frac{1}{\sqrt{2\pi}}-\sqrt{\alpha}_x}$$
$$+\frac{1}{\sqrt{2\pi}}\alpha_p^2\sqrt{\alpha}_x\frac{(1-\lambda)\phi(1)}{(\alpha_p(1-\lambda)\frac{1}{\sqrt{2\pi}}-\sqrt{\alpha}_x)^3}. \tag{4.2.4}$$

We also require $p(\tilde{p})_{aux}$ to be weakly above the sufficient locus $p(\tilde{p})_{suf}$ defined by,

$$p(\tilde{p})_{suf} - \tilde{p} = \frac{(2D-B)\phi(\frac{\bar{\theta}(\tilde{p})-\tilde{p}}{\sigma_p})\frac{d\bar{\theta}(\tilde{p})}{d\tilde{p}}\frac{1}{\sigma_p}}{B^2\eta - \phi(1)\frac{d\bar{\theta}(\tilde{p})}{d\tilde{p}}\frac{1}{\sigma_p^2}2D}, \quad \forall \tilde{p} \leq \tilde{p}^*,$$

i.e.,

$$\frac{1}{(\frac{1+\frac{B}{2D}}{2}-\frac{B}{2D})2\eta D} \leq \frac{2D-B}{B^2\eta - \phi(1)\frac{d\bar{\theta}(\tilde{p})}{d\tilde{p}}\frac{2D}{\sigma_p^2}}, \quad \forall \tilde{p} \leq \tilde{p}^*.$$

As $\frac{d\bar{\theta}(\tilde{p})}{d\tilde{p}} \geq \frac{\alpha_p}{\alpha_p - \frac{\sqrt{2\pi\alpha_x}}{1-\lambda}}$, we require,

$$\frac{1}{(\frac{1+\frac{B}{2D}}{2} - \frac{B}{2D})2\eta D} \leq \frac{2D-B}{B^2\eta - \phi(1)\frac{\alpha_p}{\alpha_p - \frac{\sqrt{2\pi\alpha_x}}{1-\lambda}}\frac{2D}{\sigma_p^2}}. \tag{4.2.5}$$

$B = \frac{w\alpha_p}{\gamma}, D = \frac{w(\alpha_x+\alpha_p)}{\gamma}$, and define $\kappa = \frac{\alpha_x}{\alpha_p} = \frac{\gamma^2\sigma_\varepsilon^2}{\omega^2\alpha_x}$, (4.2.4)(4.2.5) are reformulated as,

$$\phi(1)(\frac{\alpha_x(1-\lambda)}{\alpha_x(1-\lambda)-\kappa\sqrt{2\pi\alpha_x}} - 1)\frac{\alpha_x(1-\lambda)}{\alpha_x(1-\lambda)-\kappa\sqrt{2\pi\alpha_x}} - \frac{1}{\sqrt{2\pi}}\frac{\kappa^{\frac{3}{2}}\alpha_x^2\phi(1)(1-\lambda)}{(\frac{\alpha_x(1-\lambda)}{\sqrt{2\pi}} - \kappa\sqrt{\alpha_x})^3}$$

$$\leq (\frac{1}{4(1+\kappa)} - \frac{1}{2})^2(1+\kappa)2\eta\frac{\omega}{\gamma}, \tag{4.2.4$'$}$$

$$1-\lambda \leq \frac{(2\kappa^2+2\kappa-\frac{1}{2})\kappa\eta\frac{\omega}{\gamma}}{\phi(1)2(1+\kappa)+(2\kappa^2+2\kappa-\frac{1}{2})\eta\frac{\omega}{\gamma}}\sqrt{\frac{2\pi}{\alpha_x}}. \tag{4.2.5$'$}$$

For any $\alpha_x, \omega, \gamma, \lambda, \eta$, when κ is relatively large, condition $(4.2.4)', (4.2.5)'$ are satisfied. That implies when σ_ε is relatively large, condition $(4.2.4)', (4.2.5)'$ are satisfied. Then given any $(\tilde{p}^*, p_0)$ such that

$$\frac{\phi(\frac{\bar{\theta}(\tilde{p}^*)-\tilde{p}^*}{\sigma p})\frac{d\bar{\theta}(\tilde{p})}{d\tilde{p}}|_{\tilde{p}=\tilde{p}^*}}{(\frac{1}{2}-\frac{B}{4D})2\eta D\sigma_p} \leq p_0 - \tilde{p}^* \leq 0,$$

$p(\tilde{p})$ pinned down by (2.10) and $(\tilde{p}^*, p_0)$ satisfies the condition in Lemma 10.

■

Lemma 12. *In equilibrium $\tilde{p}^*$, $U(M,\tilde{p})|_{M=(B-D)\tilde{p}^*}$ is continuous at $\tilde{p} = \tilde{p}^*$. $(\tilde{p}^*, p_0)$ satisfies,*

$$\eta(\frac{M}{B-D} - p_0)(M + Dp_0 - B\tilde{p}^*) + \Phi(\frac{\bar{\theta}(\tilde{p}^*) - \frac{M}{B-D}}{\sigma_p}) - t =$$
$$\eta(\frac{M}{B-D} - \tilde{p}^*)(M + D\tilde{p}^* - B\tilde{p}^*)|_{M=(B-D)\tilde{p}^*},$$

thus

$$\tilde{p}^* - p_0 = \sqrt{\frac{\Phi(\frac{\bar{\theta}(\tilde{p}^*)-\tilde{p}^*}{\sigma_p}) - t}{\eta D}}.$$

Lemma 13. *In equilibrium $\tilde{p}^*$, $U(M,\tilde{p})|_{M\neq(B-D)\tilde{p}^*}$ is not continuous at $\tilde{p} = \tilde{p}^*$.*

$$\eta(\frac{M}{B-D} - \tilde{p}^*)(M + D\tilde{p}^* - B\tilde{p}^*)$$

$$\begin{cases} > \eta(\frac{M}{B-D} - p_0)(M + Dp_0 - B\tilde{p}^*) + \Phi(\frac{\bar{\theta}(\tilde{p}^*)-\frac{M}{B-D}}{\sigma_p}) - t \;\; if \;\frac{M}{B-D} > \tilde{p}^*; \\ < \eta(\frac{M}{B-D} - p_0)(M + Dp_0 - B\tilde{p}^*) + \Phi(\frac{\bar{\theta}(\tilde{p}^*)-\frac{M}{B-D}}{\sigma_p}) - t \;\; if \;\frac{M}{B-D} < \tilde{p}^*. \end{cases}$$

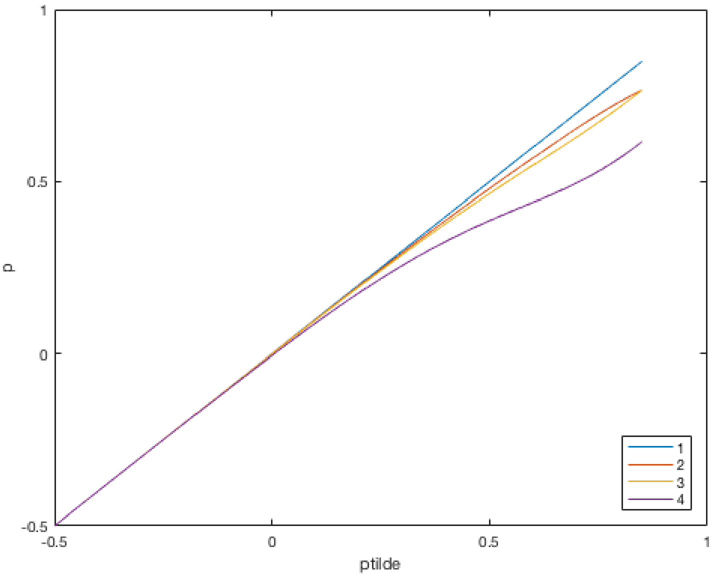

Figure 4.1: Price Locus in Lemma 11

Parameters: $\sigma_x = 0.5, \sigma_\varepsilon = 1, \omega = 1, \gamma = 2, t = 0.5, \eta = 1, \lambda = 0.7$.

Line 1 denotes $p = \tilde{p}$, line 2 denotes $p = p(\tilde{p})$ pinned down by (2.10) with initial point $(\tilde{p}^* = 0.85, p_0 = 0.7663)$, line 3 denotes $p(\tilde{p})_{aux}$, line 4 denotes $p(\tilde{p})_{suf}$.

Proof.

$$G(M) := \eta(\frac{M}{B-D} - \tilde{p}^*)(M + D\tilde{p}^* - B\tilde{p}^*) - (\eta(\frac{M}{B-D} - p_0)(M + Dp_0 - B\tilde{p}^*)$$

$$+\Phi(\frac{\bar{\theta}(\tilde{p}^*) - \frac{M}{B-D}}{\sigma_p}) - t).$$

$G(M)|_{M=(B-D)\tilde{p}^*} = 0$, $\frac{dG(M)}{dM} = (p_0 - \tilde{p}^*)\frac{2D-B}{D-B} + \phi(\frac{\bar{\theta}(\tilde{p}^*) - \frac{M}{B-D}}{\sigma_p})\frac{1}{(B-D)\sigma_p} < 0$.

■

Now we can use the above lemmas to prove Proposition 12.

Proof.

The large trader's utility function can be written as,

$$U(M, \tilde{p}) = \begin{cases} \eta(\frac{M}{B-D} - p(\tilde{p}))(M + Dp(\tilde{p}) - B\tilde{p}) + \Phi(\frac{\bar{\theta}(\tilde{p}) - \frac{M}{B-D}}{\sigma_p}) - t & \text{if } \tilde{p} \le \tilde{p}^*; \\ \eta(\frac{M}{B-D} - p(\tilde{p}))(M + Dp(\tilde{p}) - B\tilde{p}) & \text{if } \tilde{p} > \tilde{p}^*. \end{cases}$$

where $p(\tilde{p})$ is given in Proposition 2.

From (2.11) and Lemma 12, we can see the condition in Lemma 10 is satisfied, thus $\frac{\partial^2 U(M,\tilde{p})}{\partial \tilde{p} \partial M} \leq 0 \ \forall \tilde{p} \leq \tilde{p}^*, \forall M \geq (B-D)\tilde{p}^*$.

Thus $\forall (M, M')$ such that $\frac{M}{B-D} \leq \tilde{p}^*$, we have

$$\frac{\partial U(M',\tilde{p})}{\partial \tilde{p}}|_{\tilde{p}=\frac{M}{B-D}} \leq 0, \text{ if } M' > M; \frac{\partial U(M',\tilde{p})}{\partial \tilde{p}}|_{\tilde{p}=\frac{M}{B-D}} \geq 0, \text{ if } M' < M. \tag{4.2.7}$$

(1) It's easy to see $\forall M' < (B-D)\tilde{p}^*$, $\arg\max_{\tilde{p} > \tilde{p}^*} U(M', \tilde{p}) = \frac{M'}{B-D}$. Now we prove $\frac{M'}{B-D}$ is also the global maximizer.

Suppose not, i.e., $\exists \frac{M}{B-D} \leq \tilde{p}^*$ such that $U(M',\tilde{p})|_{\tilde{p}=\frac{M'}{B-D}} < U(M',\tilde{p})|_{\tilde{p}=\frac{M}{B-D}}$, together with Lemma 13, we have $\frac{\partial U(M',\tilde{p})}{\partial \tilde{p}}|_{\exists \tilde{p} \in [\frac{M}{B-D}, \frac{M'}{B-D}]} < 0$.

As $\frac{\partial U(M',\tilde{p})}{\partial \tilde{p}}|_{\tilde{p} \in (\tilde{p}^*, \frac{M'}{B-D}]} = 2(D-B)(\frac{M'}{B-D} - \tilde{p}) \geq 0$, we have $\frac{\partial U(M',\tilde{p})}{\partial \tilde{p}}|_{\exists \tilde{p} \in [\frac{M}{B-D}, \tilde{p}^*]} < 0$, contradicting (4.2.7).

(2) We have shown for any $M \geq (B-D)\tilde{p}^*$, $\arg\max_{\tilde{p} \leq \tilde{p}^*} U(M, \tilde{p}) = \frac{M}{B-D}$. Now we prove $\frac{M}{B-D}$ is also the global maximizer.

Suppose not, i.e., $\exists \frac{M'}{B-D} > \tilde{p}^*$ such that $U(M,\tilde{p})|_{\tilde{p}=\frac{M}{B-D}} < U(M,\tilde{p})|_{\tilde{p}=\frac{M'}{B-D}}$, together with Lemma 13, we have $\frac{\partial U(M,\tilde{p})}{\partial \tilde{p}}|_{\exists \tilde{p} \in [\frac{M}{B-D}, \frac{M'}{B-D}]} > 0$.

As $\frac{\partial U(M,\tilde{p})}{\partial \tilde{p}}|_{\tilde{p} \in (\tilde{p}^*, \frac{M'}{B-D}]} = 2(D-B)(\frac{M}{B-D} - \tilde{p}) \leq 0$, we have $\frac{\partial U(M,\tilde{p})}{\partial \tilde{p}}|_{\exists \tilde{p} \in [\frac{M}{B-D}, \tilde{p}^*]} > 0$, contradicting (4.2.7).

■

Proof of Proposition 13

Proof.

There are multiple $\tilde{p}^*$ that satisfy the condition in Proposition 12,

$$-\sqrt{\frac{\Phi(\frac{\bar{\theta}(\tilde{p}^*)-\tilde{p}^*}{\sigma_p}) - t}{\eta D}} \geq \frac{\phi(\frac{\bar{\theta}(\tilde{p}^*)-\tilde{p}^*}{\sigma p})\frac{d\bar{\theta}(\tilde{p})}{d\tilde{p}}|_{\tilde{p}=\tilde{p}^*}}{(\frac{1}{2} - \frac{B}{4D})2\eta D \sigma_p}. \tag{2.11}$$

Since $-\sqrt{\frac{\Phi(\frac{\bar{\theta}(\tilde{p}^*)-\tilde{p}^*}{\sigma_p})-t}{\eta D}}$ is continuous and decreasing at any $\tilde{p}^* \in [\tilde{p}_2, \tilde{p}_1]$, we have

$$-\sqrt{\frac{\Phi(\frac{\bar{\theta}(\tilde{p}^*)-\tilde{p}^*}{\sigma_p}) - t}{\eta D}}|_{\tilde{p}^*=\tilde{p}_1} = 0,$$

$$\frac{\phi(\frac{\bar{\theta}(\tilde{p}^*)-\tilde{p}^*}{\sigma p})\frac{d\bar{\theta}(\tilde{p})}{d\tilde{p}}|_{\tilde{p}=\tilde{p}^*}}{(\frac{1}{2}-\frac{B}{4D})2\eta D\sigma_p} < 0, \ \ \forall \tilde{p}^* \in [\tilde{p}_2, \tilde{p}_1].$$

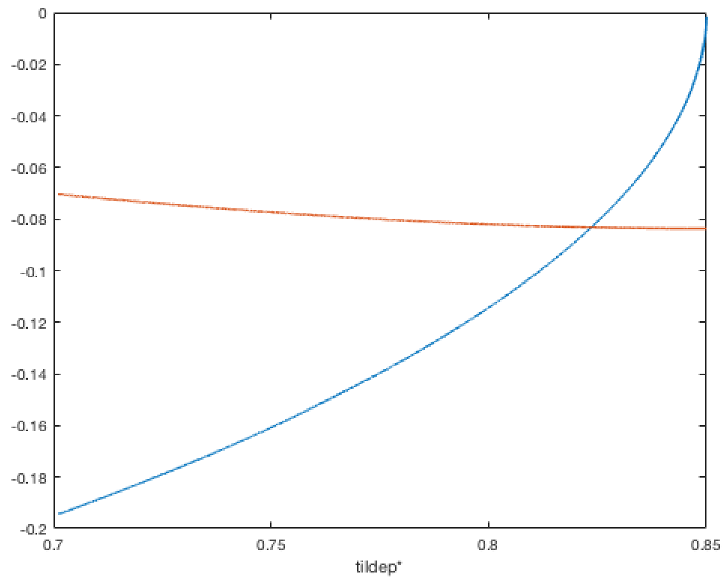

Figure 4.2: Multiple $\tilde{p}^*$ Satisfy the Condition in Proposition 12

Parameters: $\sigma_x = 0.5$, $\sigma_\varepsilon = 1$, $\omega = 1$, $\gamma = 2$, $t = 0.5$, $\eta = 1$, $\lambda = 0.7$, $p_1 = 0.85$, $p_2 = 0.7011$.

The blue line denotes the LHS of (2.11), the orange line denotes the RHS of (2.11).

∎

Proof of Proposition 14

Proof.

From local condition, $\frac{\partial^2 U(M,\tilde{p})}{\partial \tilde{p}^2}|_{\tilde{p}=\frac{M}{B-D}} \leq 0 \ \forall M \geq (B-D)\tilde{p}^*$, we have $\frac{\partial^2 U(M,\tilde{p})}{\partial \tilde{p}^2}|_{\tilde{p}=\frac{M}{B-D}} = \frac{\phi(\frac{\bar{\theta}(\tilde{p})-\tilde{p}}{\sigma_p})\frac{d\bar{\theta}(\tilde{p})}{d\tilde{p}}}{(p(\tilde{p})-\tilde{p})\sigma_p}(\frac{B}{2D}-1) + \frac{B^2\eta}{2D} \leq 0$, thus $p(\tilde{p}) - \tilde{p} \geq \phi(\frac{\bar{\theta}(\tilde{p})-\tilde{p}}{\sigma_p})\frac{d\bar{\theta}(\tilde{p})}{d\tilde{p}}\frac{2D-B}{B^2\eta\sigma_p} \ \ \forall \tilde{p} \leq \tilde{p}^*$.

Define $\eta(\tilde{p}^*)$ by

$$-\sqrt{\frac{\Phi(\frac{\bar{\theta}(\tilde{p}^*)-\tilde{p}^*}{\sigma_p}) - t}{\eta(\tilde{p}^*)D}} = \phi(\frac{\bar{\theta}(\tilde{p}^*)-\tilde{p}^*}{\sigma_p})\frac{d\bar{\theta}(\tilde{p})}{d\tilde{p}}|_{\tilde{p}=\tilde{p}^*}\frac{2D-B}{B^2\eta(\tilde{p}^*)\sigma_p}.$$

If $\eta > \eta(\tilde{p}^*)$, $\tilde{p}^*$ is not an equilibrium.

∎

Proof of Proposition 16

Proof.

Define p_3,p_4 by

$$\Phi(\frac{1-p_3}{\sigma_p}) - t = 0,$$

$$\Phi(\frac{\lambda-p_4}{\sigma_p}) - t = 0.$$

We have $p_4 < p_2 < p_1 < p_3$. Since our elimination procedure differs for $\tilde{p}$ within different ranges, our exposition is thus case by case.

1. $p_4 \leq \tilde{p} < p_1$.

1st step:

For the large trader, $\{(z,0)|z \neq 0\}$ are dominated by (0,0).

Define z^* by

$$\frac{z^{*2}\gamma}{\omega\alpha}\eta = \Phi(\frac{1-\tilde{p}}{\sigma_p}) - t,$$

thus $\{(z,1)|z \notin [-z^*, z^*]\}$ are dominated by (0,0).

Figure 4.3 depicts the large trader's expected payoff in the second-stage currency-attack game, with red line denoting his expected payoff by taking strategy $\{(z,1)|z \neq 0\}$, while blue line denoting the expected payoff by taking strategy (0,1), in the first step.

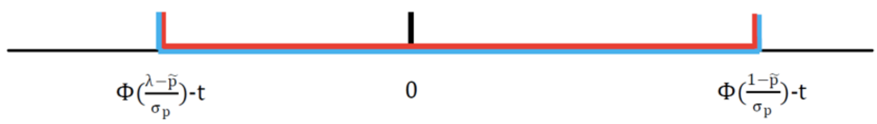

Figure 4.3: The Large Trader's Expected Payoff in the Second-stage Currency-attack Game in the First Step

In the later context, for the sake of brevity, x_1 is short for $x_1(\tilde{p})$, the strategy of small trader after observing $z = 0$ and $\tilde{p}$; x_2 is short for $x_2(z, \tilde{p})$, the strategy of small trader after observing $z \neq 0$ and $\tilde{p}$.

For any small trader, $\{(x_1, x_2|x_1, x_2 \notin [-\frac{\alpha_p}{\alpha_x}\tilde{p} - \frac{\sqrt{\alpha}}{\alpha_x}\Phi^{-1}(t), \frac{\alpha}{\alpha_x} - \frac{\alpha_p}{\alpha_x}\tilde{p} - \frac{\sqrt{\alpha}}{\alpha_x}\Phi^{-1}(t)]\}$ are eliminated, as $(\frac{\alpha}{\alpha_x} - \frac{\alpha_p}{\alpha_x}\tilde{p} - \frac{\sqrt{\alpha}}{\alpha_x}\Phi^{-1}(t))$ is the optimal threshold for the most optimistic small trader, who believes the large trader will attack and all other small traders will attack, as well; and $(-\frac{\alpha_p}{\alpha_x}\tilde{p} - \frac{\sqrt{\alpha}}{\alpha_x}\Phi^{-1}(t))$ is the optimal threshold for the most pessimistic small trader, who believes the large trader won't attack and all other small traders won't attack, as well.

2nd step:

Define $x_{2h}^{(n)}$ and $x_{2l}^{(n)}$ as the highest and lowest threshold of small traders contingent on observing the large trader participated in the first-stage asset market, after n rounds of elimination in this step.

Define $x_{1h}^{(n)}$ and $x_{1l}^{(n)}$ as the highest and lowest threshold of small traders contingent on observing the large trader did not participate in the first-stage asset market, after n rounds of elimination in this step.

At the beginning of this step, $x_{2h}^{(0)}$,$x_{1h}^{(0)}$,$x_{2l}^{(0)}$ and $x_{1l}^{(0)}$ are defined by

$$x_{2h}^{(0)} = x_{1h}^{(0)} := \frac{\alpha}{\alpha_x} - \frac{\alpha_p}{\alpha_x}\tilde{p} - \frac{\sqrt{\alpha}}{\alpha_x}\Phi^{-1}(t),$$

$$x_{2l}^{(0)} = x_{1l}^{(0)} := -\frac{\alpha_p}{\alpha_x}\tilde{p} - \frac{\sqrt{\alpha}}{\alpha_x}\Phi^{-1}(t).$$

Correspondingly, let's define $\theta_{2h}^{(0)}$, $\theta_{2l}^{(0)}$, $\theta_{1h}^{(0)}$, $\theta_{1l}^{\prime(0)}$ by

$$\lambda + (1-\lambda)\Phi(\frac{x^{(0)} - \theta^{(0)}}{\sigma_x}) = \theta^{(0)},$$

and define $\theta_{1l}^{(0)}$ by

$$(1-\lambda)\Phi(\frac{x_{1l}^{(0)} - \theta_{1l}^{(0)}}{\sigma_x}) = \theta_{1l}^{(0)}.$$

It's immediately clear that $\theta_{1l}^{(0)} < \theta_{1l}^{\prime(0)} < \theta_{2l}^{(0)} < \theta_{1h}^{(0)} = \theta_{2h}^{(0)}$.

Figure 4.4 depicts the large trader's expected payoff in the second-stage currency-attack game, with red line denoting his expected payoff by taking strategy $\{(z,1)|z \neq 0\}$, while blue line denoting the expected payoff by taking strategy (0,1), at the beginning of the second step.

Define positive value $z^{*(0)}$ by

$$\frac{{z^{*(0)}}^2\gamma}{\omega\alpha}\eta = \Phi(\frac{\theta_{2h}^{(0)} - \tilde{p}}{\sigma_p}) - t.$$

For the large trader,$\{(z,1)|z \notin [-z^{*(0)}, z^{*(0)}]\}$ are dominated by (0,0).

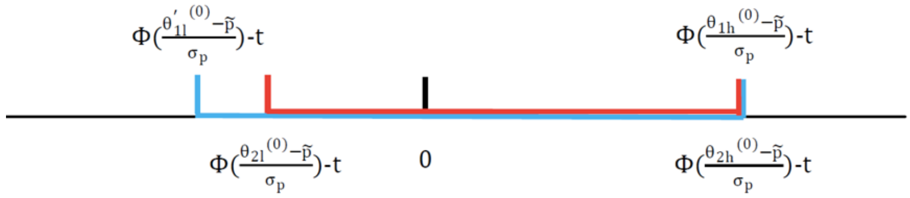

Figure 4.4: The Large Trader's Expected Payoff in the Second-stage Currency-attack Game in the Second Step

For any small trader, after observing the large trader has participated in the first-stage asset market, he believes other small traders' threshold is between $x_{2l}^{(0)}$ and $x_{2h}^{(0)}$, and the large trader will attack the currency regime . Then we could pin down $x_{2l}^{(1)}$ and $x_{2h}^{(1)}$, and we have $x_{2l}^{(0)} < x_{2l}^{(1)} < x_{2h}^{(1)} < x_{2h}^{(0)}$.

For any small trader, after observing the large trader did not participate in the first-stage asset market, he believes other small traders' threshold is between $x_{1l}^{(0)}$ and $x_{1h}^{(0)}$. Then we could pin down $x_{1l}^{(1)}$ (contingent on the large trader will not attack) and $x_{1h}^{(1)}$ (contingent on the large trader will attack), and we have $x_{1l}^{(0)} < x_{1l}^{(1)} < x_{1h}^{(1)} < x_{1h}^{(0)}$.

It's immediately clear that $x_{1l}^{(1)} < x_{2l}^{(1)} < x_{1h}^{(1)} = x_{2h}^{(1)}$.

Correspondingly, let's define $\theta_{2h}^{(1)}$, $\theta_{2l}^{(1)}$, $\theta_{1h}^{(1)}$, $\theta_{1l}^{\prime(1)}$, and $\theta_{1l}^{(1)}$, in the same way as above.

We have

$$\theta_{2l}^{(0)} < \theta_{2l}^{(1)} < \theta_{2h}^{(1)} < \theta_{2h}^{(0)},$$

$$\theta_{1l}^{(0)} < \theta_{1l}^{(1)} < \theta_{1h}^{(1)} < \theta_{1h}^{(0)},$$

$$\theta_{1l}^{(1)} < \theta_{1l}^{\prime(1)} < \theta_{2l}^{(1)} < \theta_{1h}^{(1)} = \theta_{2h}^{(1)}.$$

Next, $n = 1$.

Define positive value $z^{*(1)}$ by

$$\frac{{z^{*(1)}}^2\gamma}{\omega\alpha}\eta = \Phi(\frac{\theta_{2h}^{(1)} - \tilde{p}}{\sigma_p}) - t.$$

For the large trader, $\{(z,1)|z \notin [-z^{*(1)}, z^{*(1)}]\}$ are dominated by (0,0).

For any small trader, his belief is updated, and we could pin down $x_{2l}^{(2)}$, $x_{2h}^{(2)}$, $x_{1l}^{(2)}$, $x_{1h}^{(2)}$.

......

Until

$$\Phi(\frac{\theta_{2l}^{n^*-1} - \tilde{p}}{\sigma_p}) - t \leq 0,$$

$$\Phi(\frac{\theta_{2l}^{n^*} - \tilde{p}}{\sigma_p}) - t > 0.$$

This n^* must exist, since $\Phi(\frac{\bar{\theta}(\tilde{p})-\tilde{p}}{\sigma_p}) - t > 0$, for $\tilde{p} < p_1$ and $\lim_{n\to\infty} \theta_{2l}^{(n)} = \bar{\theta}(\tilde{p})$. For the large trader, pick z with small enough magnitude, such that $\Phi(\frac{\theta_{2l}^{n^*}-\tilde{p}}{\sigma_p}) - t - \frac{z^2\gamma}{\omega\alpha}\eta > 0$. $(z, 1)$ dominates (0,0). (0,0) is eliminated.

3rd step:

At the beginning of this step, $x_{1l}^{(n^*+1)} < x_{2l}^{(n^*+1)} < x_{1h}^{(n^*+1)} = x_{2h}^{(n^*+1)}$.

Correspondingly, by using $\lambda + (1-\lambda)\Phi(\frac{x^{(n^*+1)}-\theta^{(n^*+1)}}{\sigma_x}) = \theta^{(n^*+1)}$, we have

$$\theta_{1l}^{(n^*+1)} = \theta_{1l}'^{(n^*+1)} < \theta_{2l}^{(n^*+1)} < \theta_{1h}^{(n^*+1)} = \theta_{2h}^{(n^*+1)}.$$

Figure 4.5 depicts the large trader's expected payoff in the second-stage currency-attack game, with red line denoting his expected payoff by taking strategy $\{(z, 1)|z \neq 0\}$, while blue line denoting the expected payoff by taking strategy (0,1), at the beginning of the third step.

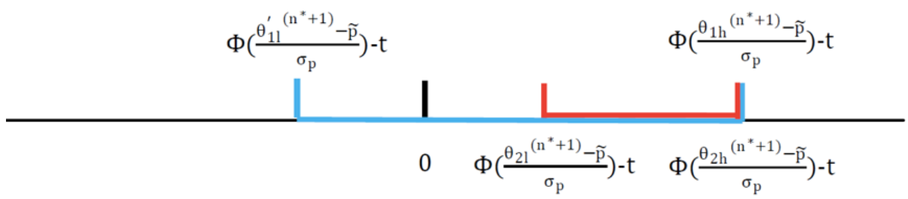

Figure 4.5: The Large Trader's Expected Payoff in the Second-stage Currency-attack Game in the Third Step

Define a positive value $z^{*(n^*+1)}$ by

$$\frac{z^{*(n^*+1)2}\gamma}{\omega\alpha}\eta = \Phi(\frac{\theta_{2h}^{(n^*+1)} - \tilde{p}}{\sigma_p}) - t - (\Phi(\frac{\theta_{2l}^{(n^*+1)} - \tilde{p}}{\sigma_p}) - t).$$

For the large trader, $\{(z, 1)|z \notin [-z^{*(n^*+1)}, z^{*(n^*+1)}]\}$ are dominated by $(\delta, 1)$, where δ is with very small magnitude such that,

$$\Phi(\frac{\theta_{2l}^{(n^*+1)} - \tilde{p}}{\sigma_p}) - t - \frac{\delta^2\gamma}{\omega\alpha}\eta > \Phi(\frac{\theta_{2h}^{(n^*+1)} - \tilde{p}}{\sigma_p}) - t - \frac{z^2\gamma}{\omega\alpha}\eta.$$

As δ converges to 0, the large trader's strategies that survive infinite rounds of elimination are (0,1).

For small traders,

$$\lim_{n\to\infty} x_{1l}^{(n)} = \lim_{n\to\infty} x_{1h}^{(n)} = \bar{x}^*, \lim_{n\to\infty} x_{2l}^{(n)} = \lim_{n\to\infty} x_{2h}^{(n)} = \bar{x}^*$$

Figure 4.6 depicts the large trader's expected payoff in the second-stage currency-attack game, after infinite rounds of elimination.

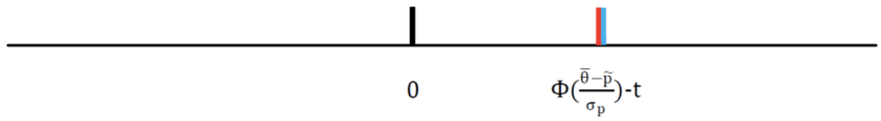

Figure 4.6: The Large Trader's Expected Payoff in the Second-stage Currency-attack Game after Infinite Rounds of Elimination

2. $\tilde{p} = p_1$.

1st step:

For the large trader, $\{(z,0)|z \neq 0\}$ are dominated by (0,0).

Define z^* by

$$\frac{z^{*2}\gamma}{\omega\alpha}\eta = \Phi(\frac{1-\tilde{p}}{\sigma_p}) - t > 0,$$

thus $\{(z,1)|z \notin [-z^*, z^*]\}$ are dominated by (0,0).

For any small trader, $\{(x_1, x_2|x_1, x_2 \notin [-\frac{\alpha_p}{\alpha_x}\tilde{p} - \frac{\sqrt{\alpha}}{\alpha_x}\Phi^{-1}(t), \frac{\alpha}{\alpha_x} - \frac{\alpha_p}{\alpha_x}\tilde{p} - \frac{\sqrt{\alpha}}{\alpha_x}\Phi^{-1}(t)]\}$ are eliminated.

2nd step: At the beginning of this step, $x_{2h}^{(0)}$, $x_{1h}^{(0)}$, $x_{2l}^{(0)}$ and $x_{1l}^{(0)}$ are defined by

$$x_{2h}^{(0)} = x_{1h}^{(0)} := \frac{\alpha}{\alpha_x} - \frac{\alpha_p}{\alpha_x}\tilde{p} - \frac{\sqrt{\alpha}}{\alpha_x}\Phi^{-1}(t),$$

$$x_{2l}^{(0)} = x_{1l}^{(0)} := -\frac{\alpha_p}{\alpha_x}\tilde{p} - \frac{\sqrt{\alpha}}{\alpha_x}\Phi^{-1}(t).$$

After this round of elimination, $x_{2l}^{(0)} < x_{2l}^{(1)} < x_{2h}^{(1)} < x_{2h}^{(0)}$, $x_{1l}^{(0)} < x_{1l}^{(1)} < x_{1h}^{(1)} < x_{1h}^{(0)}$, $x_{1l}^{(1)} < x_{2l}^{(1)} < x_{1h}^{(1)} = x_{2h}^{(1)}$.

Define $z^{*(0)}$ by

$$\frac{{z^{*(0)}}^2\gamma}{\omega\alpha}\eta = \Phi(\frac{\theta_{2h}^{(0)} - \tilde{p}}{\sigma_p}) - t.$$

For the large trader, $\{(z,1)|z \notin [-z^{*(0)}, z^{*(0)}]\}$ are dominated by (0,0).

The large trader's strategies that survive infinite rounds of elimination are (0,0) and (0,1).

For small traders,

$$\lim_{n\to\infty} x_{2l}^{(n)} = \lim_{n\to\infty} x_{2h}^{(n)} = \bar{x}^*.$$

3. $\tilde{p} < p_4$.

1st step:

For the large trader, $\{(z,0)|z \neq 0\}$ and (0,0) are dominated by (0,1), since

$$\Phi(\frac{\lambda - \tilde{p}}{\sigma_p}) - t > 0.$$

Also, define z^* by

$$\frac{z^{*2}\gamma}{\omega\alpha}\eta = \Phi(\frac{1-\tilde{p}}{\sigma_p}) - t - (\Phi(\frac{\lambda - \tilde{p}}{\sigma_p}) - t),$$

thus $\{(z,1)|z \notin [-z^*, z^*]\}$ are dominated by (0,1).

For any small trader, $\{(x_1, x_2|x_1, x_2 \notin [-\frac{\alpha_p}{\alpha_x}\tilde{p} - \frac{\sqrt{\alpha}}{\alpha_x}\Phi^{-1}(t), \frac{\alpha}{\alpha_x} - \frac{\alpha_p}{\alpha_x}\tilde{p} - \frac{\sqrt{\alpha}}{\alpha_x}\Phi^{-1}(t)]\}$ are eliminated.

2nd step:

For small traders,

$$\lim_{n\to\infty} x_{1l}^{(n)} = \lim_{n\to\infty} x_{1h}^{(n)} = \bar{x}^*, \lim_{n\to\infty} x_{2l}^{(n)} = \lim_{n\to\infty} x_{2h}^{(n)} = \bar{x}^*.$$

4. $p_1 < \tilde{p} < p_3$.

1st step: For the large trader, $\{(z,0)|z \neq 0\}$ are dominated by (0,0).

Define z^* by

$$\frac{z^{*2}\gamma}{\omega\alpha}\eta = \Phi(\frac{1-\tilde{p}}{\sigma_p}) - t > 0,$$

thus $\{(z,1)|z \notin [-z^*, z^*]\}$ are dominated by (0,0).

For any small trader, $\{(x_1, x_2|x_1, x_2 \notin [-\frac{\alpha_p}{\alpha_x}\tilde{p} - \frac{\sqrt{\alpha}}{\alpha_x}\Phi^{-1}(t), \frac{\alpha}{\alpha_x} - \frac{\alpha_p}{\alpha_x}\tilde{p} - \frac{\sqrt{\alpha}}{\alpha_x}\Phi^{-1}(t)]\}$ are eliminated.

2nd step:

At the beginning of this step, $x_{2h}^{(0)}$, $x_{1h}^{(0)}$, $x_{2l}^{(0)}$ and $x_{1l}^{(0)}$ are defined by

$$x_{2h}^{(0)} = x_{1h}^{(0)} := \frac{\alpha}{\alpha_x} - \frac{\alpha_p}{\alpha_x}\tilde{p} - \frac{\sqrt{\alpha}}{\alpha_x}\Phi^{-1}(t),$$

$$x_{2l}^{(0)} = x_{1l}^{(0)} := -\frac{\alpha_p}{\alpha_x}\tilde{p} - \frac{\sqrt{\alpha}}{\alpha_x}\Phi^{-1}(t).$$

After this round of elimination, $x_{2l}^{(0)} < x_{2l}^{(1)} < x_{2h}^{(1)} < x_{2h}^{(0)}$, $x_{1l}^{(0)} < x_{1l}^{(1)} < x_{1h}^{(1)} < x_{1h}^{(0)}$, $x_{1l}^{(1)} < x_{2l}^{(1)} < x_{1h}^{(1)} = x_{2h}^{(1)}$.

Define $z^{*(0)}$ by

$$\frac{{z^{*(0)}}^2\gamma}{\omega\alpha}\eta = \Phi(\frac{\theta_{2h}^{(0)} - \tilde{p}}{\sigma_p}) - t.$$

For the large trader, $\{(z,1)|z \notin [-z^{*(0)}, z^{*(0)}]\}$ are dominated by (0,0).

......

Until

$$\Phi(\frac{\theta_{2h}^{n^*-1} - \tilde{p}}{\sigma_p}) - t \geq 0,$$

$$\Phi(\frac{\theta_{2h}^{n^*} - \tilde{p}}{\sigma_p}) - t < 0.$$

This n^* must exist.For the large trader, $\{(z,1)|z \neq 0\}$ and (0,1) are dominated by (0,0), and the only strategy surviving n^* rounds of elimination is (0,0).

For small traders,

$$\lim_{n\to\infty} x_{1l}^{(n)} = \underline{x}^*, \lim_{n\to\infty} x_{1h}^{(n)} = \underline{x}^*.$$

5. $\tilde{p} = p_3$.

1st step:

For the large trader, $\{(z,0)|z \neq 0\}$ and $\{(z,1)|z \neq 0\}$ are dominated by (0,0).

For any small trader, $\{(x_1, x_2|x_1, x_2 \notin [-\frac{\alpha_p}{\alpha_x}\tilde{p} - \frac{\sqrt{\alpha}}{\alpha_x}\Phi^{-1}(t), \frac{\alpha}{\alpha_x} - \frac{\alpha_p}{\alpha_x}\tilde{p} - \frac{\sqrt{\alpha}}{\alpha_x}\Phi^{-1}(t)]\}$ are eliminated.

2nd step:

At the beginning of this step, $x_{2h}^{(0)}$, $x_{1h}^{(0)}$, $x_{2l}^{(0)}$ and $x_{1l}^{(0)}$ are defined by

$$x_{2h}^{(0)} = x_{1h}^{(0)} := \frac{\alpha}{\alpha_x} - \frac{\alpha_p}{\alpha_x}\tilde{p} - \frac{\sqrt{\alpha}}{\alpha_x}\Phi^{-1}(t),$$

$$x_{2l}^{(0)} = x_{1l}^{(0)} := -\frac{\alpha_p}{\alpha_x}\tilde{p} - \frac{\sqrt{\alpha}}{\alpha_x}\Phi^{-1}(t).$$

After this round of elimination, $x_{1l}^{(0)} < x_{1l}^{(1)} < x_{1h}^{(1)} < x_{1h}^{(0)}$.

......

Until

$$\Phi(\frac{\theta_{1h}^{n^*-1} - \tilde{p}}{\sigma_p}) - t \geq 0,$$

$$\Phi(\frac{\theta_{1h}^{n^*} - \tilde{p}}{\sigma_p}) - t < 0.$$

This n^* must exist.

For the large trader, (0,1) is dominated by (0,0), and the only strategy surviving n^* rounds of elimination is (0,0).

For small traders,

$$\lim_{n\to\infty} x_{1l}^{(n)} = \underline{x}^*, \lim_{n\to\infty} x_{1h}^{(n)} = \underline{x}^*.$$

6. $\tilde{p} > p_3$.

1st step:

For the large trader, $\{(z,0)|z \neq 0\}$, $\{(z,1)|z \neq 0\}$, (0,1) are dominated by (0,0), since $\Phi(\frac{1-\tilde{p}}{\sigma_p}) - t < 0$.

For any small trader, $\{(x_1, x_2|x_1, x_2 \notin [-\frac{\alpha_p}{\alpha_x}\tilde{p} - \frac{\sqrt{\alpha}}{\alpha_x}\Phi^{-1}(t), \frac{\alpha}{\alpha_x} - \frac{\alpha_p}{\alpha_x}\tilde{p} - \frac{\sqrt{\alpha}}{\alpha_x}\Phi^{-1}(t)]\}$ are eliminated.

2nd step:

For small traders,

$$\lim_{n\to\infty} x_{1l}^{(n)} = \underline{x}^*, \lim_{n\to\infty} x_{1h}^{(n)} = \underline{x}^*.$$

■

4.3 Appendix of Chapter 3

4.3.1 Central Banks' Asset Allocation Problem

In this appendix, we generalize our model in the main text such that the asset returns are incorporated into consideration. Then from central banks' perspective, different assets denominated in the same foreign currency are with different returns and risks. Central banks' problem is an asset allocation problem. The model in the main text is a degenerate case that assumes away the assets' returns.

The central bank decides the share of the asset j. The return rate of asset j is r_j. $\mathbf{\Omega}$ is the covariance matrix of assets return rates, $\mathbf{m}$ is the expected return rates of assets. R_L is the growth rate of the value of the obligations using domestic currency as numeraire.

The value of the liquidity tranche after one period is

$$\sum_j Lx_{Lj}(1+r_j) = L(1+\sum_j x_{Lj}r_j).$$

The value of the investment tranche after one period is

$$\sum_j (A-L)x_{Ij}(1+r_j) = (A-L)(1+\sum_j x_{Ij}r_j).$$

The goal for the liquidity tranche is to maximize the surplus under some risk budget

$$L\sum_j x_{Lj}r_j - LR_L.$$

We have

$$Var(\sum_j x_{Lj}r_j - R_L) = \frac{1}{2}\mathbf{x}_L^\intercal \mathbf{\Omega} \mathbf{x}_L - \sum_j x_j Cov(r_j, R_L) + Var(R_L).$$

Thus the central bank's optimization problem is

$$\max_{\mathbf{x}_L} \frac{1}{2}\mathbf{x}_L^\intercal \mathbf{\Omega} \mathbf{x}_L - \gamma^\intercal \mathbf{x}_L - \lambda_L \mathbf{m}^\intercal \mathbf{x}_L,$$

s.t. $\mathbf{e}^\intercal \mathbf{x}_L = 1$, where $\gamma_j = Cov(r_j, R_L)$.

The FOC implies

$$\mathbf{\Omega}\mathbf{x}_L - \gamma - \lambda_L \mathbf{m} - \mu \mathbf{e} = 0,$$

so we have

$$\mathbf{\Omega}^{-1}(\gamma + \lambda_L \mathbf{m} + \mu \mathbf{e}) = \mathbf{x}_L,$$

$$\mathbf{e}^\intercal \mathbf{\Omega}^{-1}(\gamma + \lambda_L \mathbf{m} + \mu \mathbf{e}) = 1,$$

then

$$\mu = \frac{1 - \mathbf{e}^\intercal \mathbf{\Omega}^{-1}(\gamma + \lambda_L \mathbf{m})}{\mathbf{e}^\intercal \mathbf{\Omega}^{-1} \mathbf{e}},$$

$$\mathbf{x}_L = \mathbf{\Omega}^{-1}(\gamma + \lambda_L \mathbf{m} + \frac{1 - \mathbf{e}^\intercal \mathbf{\Omega}^{-1}(\gamma + \lambda_L \mathbf{m})}{\mathbf{e}^\intercal \mathbf{\Omega}^{-1}\mathbf{e}}\mathbf{e})$$

$$= \frac{\mathbf{\Omega}^{-1}\mathbf{e}}{\mathbf{e}^\intercal \mathbf{\Omega}^{-1}\mathbf{e}} + \lambda_L(\mathbf{\Omega}^{-1}\mathbf{m} - \frac{\mathbf{e}^\intercal \mathbf{\Omega}^{-1}\mathbf{m}}{\mathbf{e}^\intercal \mathbf{\Omega}^{-1}\mathbf{e}}\mathbf{\Omega}^{-1}\mathbf{e}) + (\mathbf{\Omega}^{-1}\gamma - \frac{\mathbf{e}^\intercal \mathbf{\Omega}^{-1}\gamma}{\mathbf{e}^\intercal \mathbf{\Omega}^{-1}\mathbf{e}}\mathbf{\Omega}^{-1}\mathbf{e}),$$

where $\frac{\mathbf{\Omega}^{-1}\mathbf{e}}{\mathbf{e}^\intercal \mathbf{\Omega}^{-1}\mathbf{e}} + (\mathbf{\Omega}^{-1}\gamma - \frac{\mathbf{e}^\intercal \mathbf{\Omega}^{-1}\gamma}{\mathbf{e}^\intercal \mathbf{\Omega}^{-1}\mathbf{e}}\mathbf{\Omega}^{-1}\mathbf{e})$ is the minimum variance portfolio ($\lambda_L = 0$).

$R_L = \sum_i y_i R_i$, $\gamma = \sum_i y_i \gamma^i$, where $\gamma^i_j = Cov(r_j, R_i)$, namely,

$$\gamma = \begin{pmatrix} Cov(r_1, R_1), Cov(r_1, R_2), ... \\ Cov(r_2, R_1), Cov(r_2, R_2), ... \\ ... \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ ... \end{pmatrix} = \mathbf{S}\mathbf{y}.$$

The model in the main context is a special case where $\mathbf{S}$ is the covariance matrix of the currencies' return rates,

$$\gamma = \begin{pmatrix} Cov(R_1, R_1), Cov(R_1, R_2), ... \\ Cov(R_2, R_1), Cov(R_2, R_2), ... \\ ... \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ ... \end{pmatrix} = \mathbf{\Omega}\mathbf{y}.$$

The goal for the investment tranche is to maximize the returns under some risk budget. It's equivalent to maximize $1 + \sum_j x_{Ij} r_j$. The central bank's optimization problem for the investment tranche is

$$\min_{\mathbf{x}_I} \frac{1}{2}\mathbf{x}_I^\intercal \mathbf{\Omega}\mathbf{x}_I - \lambda_I \mathbf{m}^\intercal \mathbf{x}_I,$$

s.t. $\mathbf{e}^\intercal \mathbf{x}_I = 1$.

The solution is

$$\mathbf{x}_I = \frac{\mathbf{\Omega}^{-1}\mathbf{e}}{\mathbf{e}^\intercal \mathbf{\Omega}^{-1}\mathbf{e}} + \lambda_I(\mathbf{\Omega}^{-1}\mathbf{m} - \frac{\mathbf{e}^\intercal \mathbf{\Omega}^{-1}\mathbf{m}}{\mathbf{e}^\intercal \mathbf{\Omega}^{-1}\mathbf{e}}\mathbf{\Omega}^{-1}\mathbf{e}).$$

If $L \geq A$, the whole portfolio is the liquidity tranche $\mathbf{x} = \mathbf{x}_L$. Otherwise

$$\mathbf{x} = \frac{L\mathbf{x}_L + (A - L)\mathbf{x}_I}{A} = \mathbf{x}_I + \frac{L}{A}(\mathbf{x}_L - \mathbf{x}_I)$$

$$= \frac{\mathbf{\Omega}^{-1}\mathbf{e}}{\mathbf{e}^\intercal \mathbf{\Omega}^{-1}\mathbf{e}} + \lambda_I(\mathbf{\Omega}^{-1}\mathbf{m} - \frac{\mathbf{e}^\intercal \mathbf{\Omega}^{-1}\mathbf{m}}{\mathbf{e}^\intercal \mathbf{\Omega}^{-1}\mathbf{e}}\mathbf{\Omega}^{-1}\mathbf{e})$$

$$+\frac{L}{A}(\mathbf{\Omega}^{-1}\gamma - \frac{\mathbf{e}^\intercal \mathbf{\Omega}^{-1}\gamma}{\mathbf{e}^\intercal \mathbf{\Omega}^{-1}\mathbf{e}}\mathbf{\Omega}^{-1}\mathbf{e} + (\lambda_L - \lambda_I)(\mathbf{\Omega}^{-1}\mathbf{m} - \frac{\mathbf{e}^\intercal \mathbf{\Omega}^{-1}\mathbf{m}}{\mathbf{e}^\intercal \mathbf{\Omega}^{-1}\mathbf{e}}\mathbf{\Omega}^{-1}\mathbf{e})).$$

The obligations of foreign currencies depend on the imports payment and short-term external debt service,

$$L\mathbf{y} = a\mathbf{T} + b\mathbf{D}.$$

The aggregate obligation and the size of the liquidity tranche are

$$L = \mathbf{e}^\intercal L\mathbf{y} = \mathbf{e}^\intercal(a\mathbf{T} + b\mathbf{D}),$$

$$\mathbf{y} = \frac{a\mathbf{T} + b\mathbf{D}}{\mathbf{e}^\intercal(a\mathbf{T} + b\mathbf{D})},$$

$$l \in \begin{cases} J_1 & \text{if } \; \mathbf{e}^\intercal(a\mathbf{T}_l + b\mathbf{D}_l) < A_l; \\ J_2 & \text{if } \; \mathbf{e}^\intercal(a\mathbf{T}_l + b\mathbf{D}_l) \geq A_l. \end{cases}$$

The asset composition of central bank $l \in J_1$ are,

$$\mathbf{x} = \frac{\mathbf{\Omega}^{-1}\mathbf{e}}{\mathbf{e}^\intercal\mathbf{\Omega}^{-1}\mathbf{e}} + \lambda_I(\mathbf{\Omega}^{-1}\mathbf{m} - \frac{\mathbf{e}^\intercal\mathbf{\Omega}^{-1}\mathbf{m}}{\mathbf{e}^\intercal\mathbf{\Omega}^{-1}\mathbf{e}}\mathbf{\Omega}^{-1}\mathbf{e}) + \frac{1}{A}(a(\mathbf{\Omega}^{-1}\mathbf{ST} - \frac{\mathbf{\Omega}^{-1}\mathbf{e}}{\mathbf{e}^\intercal\mathbf{\Omega}^{-1}\mathbf{e}}\mathbf{e}^\intercal\mathbf{\Omega}^{-1}\mathbf{ST})$$

$$+b(\mathbf{\Omega}^{-1}\mathbf{SD} - \frac{\mathbf{\Omega}^{-1}\mathbf{e}}{\mathbf{e}^\intercal\mathbf{\Omega}^{-1}\mathbf{e}}\mathbf{e}^\intercal\mathbf{\Omega}^{-1}\mathbf{SD})) + \frac{1}{A}(\lambda_L - \lambda_I)(\mathbf{\Omega}^{-1}\mathbf{m} - \frac{\mathbf{e}^\intercal\mathbf{\Omega}^{-1}\mathbf{m}}{\mathbf{e}^\intercal\mathbf{\Omega}^{-1}\mathbf{e}}\mathbf{\Omega}^{-1}\mathbf{e})\mathbf{e}^\intercal(a\mathbf{T} + b\mathbf{D}).$$

The asset composition of central bank $l \in J_2$ are,

$$\mathbf{x} = \frac{\mathbf{\Omega}^{-1}\mathbf{e}}{\mathbf{e}^\intercal\mathbf{\Omega}^{-1}\mathbf{e}} + \lambda_L(\mathbf{\Omega}^{-1}\mathbf{m} - \frac{\mathbf{e}^\intercal\mathbf{\Omega}^{-1}\mathbf{m}}{\mathbf{e}^\intercal\mathbf{\Omega}^{-1}\mathbf{e}}\mathbf{\Omega}^{-1}\mathbf{e}) + (\mathbf{\Omega}^{-1}\mathbf{Sy} - \frac{\mathbf{e}^\intercal\mathbf{\Omega}^{-1}\mathbf{Sy}}{\mathbf{e}^\intercal\mathbf{\Omega}^{-1}\mathbf{e}}\mathbf{\Omega}^{-1}\mathbf{e})$$

$$= \frac{\mathbf{\Omega}^{-1}\mathbf{e}}{\mathbf{e}^\intercal\mathbf{\Omega}^{-1}\mathbf{e}} + \lambda_L(\mathbf{\Omega}^{-1}\mathbf{m} - \frac{\mathbf{e}^\intercal\mathbf{\Omega}^{-1}\mathbf{m}}{\mathbf{e}^\intercal\mathbf{\Omega}^{-1}\mathbf{e}}\mathbf{\Omega}^{-1}\mathbf{e})$$

$$+(\mathbf{\Omega}^{-1}\mathbf{S}\frac{a\mathbf{T} + b\mathbf{D}}{\mathbf{e}^\intercal(a\mathbf{T} + b\mathbf{D})} - \frac{\mathbf{\Omega}^{-1}\mathbf{e}}{\mathbf{e}^\intercal\mathbf{\Omega}^{-1}\mathbf{e}}\mathbf{e}^\intercal\mathbf{\Omega}^{-1}\mathbf{S}\frac{a\mathbf{T} + b\mathbf{D}}{\mathbf{e}^\intercal(a\mathbf{T} + b\mathbf{D})}).$$

Aggregating the optimal choice of central banks $l \in J_1$ and $l \in J_2$, assuming each central bank has the same λ_L, λ_I, a, b,

$$\mathbf{x}_{aggregate}(\mathbf{A}, \mathbf{\Omega}, \mathbf{S}, \mathbf{m}, \mathbf{T}, \mathbf{D}, a, b, \lambda_I, \lambda_L) := \frac{\sum_{l \in J} A_l \mathbf{x}_l}{\sum_{l \in J} A_l} = \frac{\sum_{l \in J} A_l \frac{\mathbf{\Omega}_l^{-1}\mathbf{e}}{\mathbf{e}^\intercal\mathbf{\Omega}_l^{-1}\mathbf{e}}}{\sum_{l \in J} A_l}$$

$$+a\frac{\sum_{l \in J_1}(\mathbf{\Omega}_l^{-1}\mathbf{S}_l\mathbf{T}_l - \frac{\mathbf{\Omega}_l^{-1}\mathbf{e}}{\mathbf{e}^\intercal\mathbf{\Omega}_l^{-1}\mathbf{e}}\mathbf{e}^\intercal\mathbf{\Omega}_l^{-1}\mathbf{S}_l\mathbf{T}_l)}{\sum_{l \in J} A_l} + b\frac{\sum_{l \in J_1}(\mathbf{\Omega}_l^{-1}\mathbf{S}_l\mathbf{D}_l - \frac{\mathbf{\Omega}_l^{-1}\mathbf{e}}{\mathbf{e}^\intercal\mathbf{\Omega}_l^{-1}\mathbf{e}}\mathbf{e}^\intercal\mathbf{\Omega}_l^{-1}\mathbf{S}_l\mathbf{D}_l)}{\sum_{l \in J} A_l}$$

$$+\frac{\sum_{l \in J_2} A_l(\mathbf{\Omega}_l^{-1}\mathbf{S}_l\frac{a\mathbf{T}_l + b\mathbf{D}_l}{\mathbf{e}^\intercal(a\mathbf{T}_l + b\mathbf{D}_l)} - \frac{\mathbf{e}^\intercal\mathbf{\Omega}_l^{-1}\mathbf{S}_l}{\mathbf{e}^\intercal\mathbf{\Omega}_l^{-1}\mathbf{e}}\frac{a\mathbf{T}_l + b\mathbf{D}_l}{\mathbf{e}^\intercal(a\mathbf{T}_l + b\mathbf{D}_l)}\mathbf{\Omega}_l^{-1}\mathbf{e})}{\sum_{l \in J} A_l}$$

$$+\lambda_I(\frac{\sum_{l \in J_1} A_l(\mathbf{\Omega}_l^{-1}\mathbf{m}_l - \frac{\mathbf{e}^\intercal\mathbf{\Omega}_l^{-1}\mathbf{m}_l}{\mathbf{e}^\intercal\mathbf{\Omega}_l^{-1}\mathbf{e}}\mathbf{\Omega}_l^{-1}\mathbf{e})}{\sum_{l \in J} A_l} - \frac{\sum_{l \in J_1}(\mathbf{\Omega}_l^{-1}\mathbf{m}_l - \frac{\mathbf{e}^\intercal\mathbf{\Omega}_l^{-1}\mathbf{m}_l}{\mathbf{e}^\intercal\mathbf{\Omega}_l^{-1}\mathbf{e}}\mathbf{\Omega}_l^{-1}\mathbf{e})\mathbf{e}^\intercal(a\mathbf{T}_l + b\mathbf{D}_l)}{\sum_{l \in J} A_l})$$

$$+\lambda_L(\frac{\sum_{l \in J_1}(\mathbf{\Omega}_l^{-1}\mathbf{m}_l - \frac{\mathbf{e}^\intercal\mathbf{\Omega}_l^{-1}\mathbf{m}_l}{\mathbf{e}^\intercal\mathbf{\Omega}_l^{-1}\mathbf{e}}\mathbf{\Omega}_l^{-1}\mathbf{e})\mathbf{e}^\intercal(a\mathbf{T}_l + b\mathbf{D}_l)}{\sum_{l \in J} A_l} + \frac{\sum_{l \in J_2} A_l(\mathbf{\Omega}_l^{-1}\mathbf{m}_l - \frac{\mathbf{e}^\intercal\mathbf{\Omega}_l^{-1}\mathbf{m}_l}{\mathbf{e}^\intercal\mathbf{\Omega}_l^{-1}\mathbf{e}}\mathbf{\Omega}_l^{-1}\mathbf{e})}{\sum_{l \in J} A_l}).$$

As we don't observe the asset composition, I transform the asset composition into the observable currency composition. Define the assets denomination matrix $\mathbf{M}$, $M_{ij} = 1$ iff asset j is denominated in currency i. Then $\mathbf{Mx}_{aggregate}$ is the currency composition of central banks' aggregate foreign exchange reserves.

Define $\mathbf{\Lambda}_t := (\mathbf{A}_t, \mathbf{\Omega}_t, \mathbf{S}_t, \mathbf{m}_t, \mathbf{T}_t, \mathbf{D}_t)$. Assuming x_{it}, the share of currency i in developing countries' foreign exchange reserves in quarter t satisfies

$$x_{it} = c_i + \beta \mathbf{x}(\mathbf{Mx}(\mathbf{\Lambda}_t, a, b, \lambda_I, \lambda_L)_{aggregate})_i + \varepsilon_{it}.$$

Then we can use similar moment conditions as in the main text to get the GMM estimator. The estimation procedure is basically the same as what we did in the main text.

We can extend this model and do the estimation for the case that the assets choice set of the liquidity tranche is restricted to only include the short-dated securities such as T-bills, time deposits, etc.; while the investment tranche also includes the long-term securities, equities, etc.

4.3.2 Estimation of the Asymptotic Variance

Under a set of regularity conditions,[4]

$$\sqrt{n}(\hat{\theta} - \theta) \xrightarrow{d} N\Big(0, (\mathbf{G}_0^\intercal \mathbf{W}_0 \mathbf{G}_0)^{-1} \mathbf{G}_0^\intercal \mathbf{W}_0 \mathbf{S}_0 \mathbf{W}_0 \mathbf{G}_0 (\mathbf{G}_0^\intercal \mathbf{W}_0 \mathbf{G}_0)^{-1}\Big),$$

where

$$\mathbf{G}_0 = E[\frac{\partial \mathbf{h}_i(\mathbf{x}_t, \mathbf{\Lambda}_t, \theta)}{\partial \theta^\intercal}],$$

$$\mathbf{S}_0 = E[\mathbf{h}_i(\mathbf{x}_t, \mathbf{\Lambda}_t, \theta)\mathbf{h}_i(\mathbf{x}_t, \mathbf{\Lambda}_t, \theta)^\intercal],$$

$$\mathbf{W}_0 = \text{plim} \mathbf{W}_n.$$

[4]Note the sample moments are not differentiable at (a, b) due to the kinks where $\mathbf{e}^\intercal(a\mathbf{T}_{lt} + b\mathbf{D}_{lt}) = A_l$ for some l, t. To be more specific about the kinks, on one side, the change of (a, b) affects the currency composition of the liquidity tranche, and it also has the size effect as the relative size of the liquidity tranche to the investment tranche for central bank l in period t changes; one the other side, as the whole portfolio of central bank l in period t is the liquidity tranche, the size effect goes away. So we are faced with a GMM with nonsmooth moments. The regularity conditions under which the consistency and asymptotic normality can be established can be seen at Newey and McFadden (1994).

$\mathbf{G}_0, \mathbf{S}_0$ can be estimated by $\hat{\mathbf{G}}, \hat{\mathbf{S}}$, where

$$\hat{\mathbf{G}} = \frac{1}{n} \sum_{it} \frac{\partial \mathbf{h}_i(\mathbf{x}_t, \mathbf{\Lambda}_t, \hat{\theta})}{\partial \theta^\intercal},$$

$$\hat{\mathbf{S}} = \frac{1}{n} \sum_{it} \mathbf{h}_i(\mathbf{x}_t, \mathbf{\Lambda}_t, \hat{\theta}) \mathbf{h}_i(\mathbf{x}_t, \mathbf{\Lambda}_t, \hat{\theta})^\intercal.$$

$\mathbf{W}_0$ is estimated by $\mathbf{W}_n$ we calculate in the main text.[5]

$$\frac{\partial \mathbf{h}_i(\mathbf{x}_t, \mathbf{\Lambda}_t, \hat{\theta})}{\partial a} = \big(\mathbf{u}_i(\mathbf{\Lambda}_t, a, b) - \bar{\mathbf{u}}_i(\mathbf{\Lambda}_t, a, b), 1\big)\beta(-(\mathbf{u}_{i2} - \bar{\mathbf{u}}_{i2}) - (\mathbf{u}_{i4} - \bar{\mathbf{u}}_{i4}) - (\lambda_L - \lambda_I)(\mathbf{u}_{i7} - \bar{\mathbf{u}}_{i7})),$$

$$\frac{\partial \mathbf{h}_i(\mathbf{x}_t, \mathbf{\Lambda}_t, \hat{\theta})}{\partial b} = \big(\mathbf{u}_i(\mathbf{\Lambda}_t, a, b) - \bar{\mathbf{u}}_i(\mathbf{\Lambda}_t, a, b), 1\big)\beta(-(\mathbf{u}_{i3} - \bar{\mathbf{u}}_{i3}) - (\mathbf{u}_{i5} - \bar{\mathbf{u}}_{i5}) - (\lambda_L - \lambda_I)(\mathbf{u}_{i8} - \bar{\mathbf{u}}_{i8})),$$

$$\frac{\partial \mathbf{h}_i(\mathbf{x}_t, \mathbf{\Lambda}_t, \hat{\theta})}{\partial \beta} = \big(\mathbf{u}_i(\mathbf{\Lambda}_t, a, b) - \bar{\mathbf{u}}_i(\mathbf{\Lambda}_t, a, b), 1\big)$$

$$(1, a, b, a, b, \lambda_I, (\lambda_L - \lambda_I)a, (\lambda_L - \lambda_I)b, \lambda_L)(\mathbf{u}_i(\mathbf{\Lambda}_t, a, b) - \bar{\mathbf{u}}_i(\mathbf{\Lambda}_t, a, b)),$$

$$\frac{\partial \mathbf{h}_i(\mathbf{x}_t, \mathbf{\Lambda}_t, \hat{\theta})}{\partial \lambda_I} = \big(\mathbf{u}_i(\mathbf{\Lambda}_t, a, b) - \bar{\mathbf{u}}_i(\mathbf{\Lambda}_t, a, b), 1\big)(-\beta)(\mathbf{u}_{i6} - \bar{\mathbf{u}}_{i6} - a(\mathbf{u}_{i7} - \bar{\mathbf{u}}_{i7}) - b(\mathbf{u}_{i8} - \bar{\mathbf{u}}_{i8})),$$

$$\frac{\partial \mathbf{h}_i(\mathbf{x}_t, \mathbf{\Lambda}_t, \hat{\theta})}{\partial \lambda_L} = \big(\mathbf{u}_i(\mathbf{\Lambda}_t, a, b) - \bar{\mathbf{u}}_i(\mathbf{\Lambda}_t, a, b), 1\big)(-\beta)(a(\mathbf{u}_{i7} - \bar{\mathbf{u}}_{i7}) + b(\mathbf{u}_{i8} - \bar{\mathbf{u}}_{i8}) + \mathbf{u}_{i9} - \bar{\mathbf{u}}_{i9}).$$

[5]If we estimate $\mathbf{W}_0$ using $\hat{\theta}$, then our estimate of $\mathbf{W}_0$ equals to $\hat{\mathbf{S}}^{-1}$, our estimate of the asymptotic variance is $(\hat{\mathbf{G}}^\intercal \hat{\mathbf{S}}^{-1} \hat{\mathbf{G}})^{-1}$. The quantitative results are similar.

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